

Chiral expansion of baryon masses and σ -terms

B. Borasoy^{#1}, Ulf-G. Meißner^{#2#3}

Universität Bonn, Institut für Theoretische Kernphysik
Nussallee 14-16, D-53115 Bonn, Germany

Abstract

We analyze the octet baryon masses and the pion/kaon-nucleon σ -terms in the framework of heavy baryon chiral perturbation theory. We include *all* terms up-to-and-including quadratic order in the light quark masses, m_q . We develop a consistent scheme to estimate low-energy constants related to scalar-isoscalar operators in the framework of resonance exchange involving one-loop graphs. The pertinent low-energy constants can only be estimated up to some finite coefficients. Including contributions from loop graphs with intermediate spin-3/2 decuplet and spin-1/2 octet states and from tree graphs including scalar mesons, we use the octet baryon masses and the pion-nucleon σ -term to fix all but one of these coefficients. Physical results are insensitive to this remaining parameter. It is also demonstrated that two-loop corrections only modify some of the subleading low-energy constants. We find for the baryon mass in the chiral limit, $\overset{\circ}{m} = 770 \pm 110$ MeV. While the corrections of order m_q^2 are small for the nucleon, they are still sizeable for the Λ , the Σ and the Ξ . Therefore a definitive statement about the convergence of three-flavor baryon chiral perturbation can not yet be made. The strangeness content of the nucleon is $y = 0.21 \pm 0.20$. We also estimate the kaon-nucleon σ -terms and some two-loops contributions to the nucleon mass.

^{#1}email: borasoy@itkp.uni-bonn.de

^{#2}email: meissner@itkp.uni-bonn.de

^{#3} Address after Oct. 1, 1996: FZ Jülich, IKP (Theorie), D-52425 Jülich, Germany

1 Introduction

Despite two decades of calculations, there is still sizeable uncertainty about the quark mass expansion of the octet baryon masses in the framework of chiral perturbation theory, see e.g. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The penultimate goal of these calculations is to get additional bounds on the ratios of the light quark masses and to understand the success of the Gell–Mann–Okubo mass relation. Originally, it was argued that the leading non-analytic corrections of the type M_ϕ^3 , where ϕ denotes the Goldstone boson fields π, K, η , are so large that chiral perturbation theory is meaningless in this case (for the most extensive study see Gasser [3]). In the same paper, a meson–cloud model was proposed which showed a faster convergence and allowed to give some bounds on the quark mass ratios. With the advent of a heavy baryon chiral perturbation theory as proposed by Jenkins and Manohar [14], which allows for a consistent power counting and is in spirit similar to the meson–cloud model, the problem of the chiral expansion of the baryon masses was taken up again. In particular, it was argued by Jenkins [5] that the inclusion of the decuplet in the effective field theory provides a natural framework to account for the large cancellations between meson loop and contact term contributions at orders q^2 , q^3 and (partly) q^4 (where q denotes a small momentum or meson mass) thus giving credit to the Gell–Mann–Okubo (GMO) mass relation. However, the inclusion of the decuplet in that approach was not done systematically and criticized in Ref.[7].^{#4} Lebed and Luty [8] analyzed the baryon masses to second order in the quark masses, claiming that the quark mass ratios and deviations from the GMO relations are not computable, whereas the corrections to the Coleman–Glashow relations in the decuplet are. We have already pointed out in the letter [16] that in that paper not all contributions at order m_q^2 and $m_q^2 \ln m_q$ are accounted for. Furthermore, most of the existing papers considered mostly the so-called computable corrections of order m_q^2 (modulo logs) or included some of the finite terms at this order [5, 6, 8, 10]. This, however, contradicts the spirit of chiral perturbation theory (CHPT) in that all terms at a given order have to be retained, see e.g. [17, 18, 19]. In Ref.[16] we presented the results of a first calculation including *all* terms of order $\mathcal{O}(m_q^2)$, where m_q is a generic symbol for any one of the light quark masses $m_{u,d,s}$. In that paper, the isospin limit $m_u = m_d$ was taken and electromagnetic corrections were neglected. Concerning the latter, we remark here that there is a fundamental problem in that the electromagnetic interactions lead to an infinite renormalization of the quark masses as first pointed out by Gasser and Leutwyler [20]. This problem has yet to be addressed in a systematic fashion and we thus will work in a world with $\alpha = 0$, with $\alpha = e^2/4\pi$ the fine structure constant. We note that the treatment of estimating low-energy constants from intermediate decuplet states in Ref.[16] was inconsistent. Here, we will give the correct and much more detailed treatment of the s-channel resonance exchanges to the low-energy constants and thus refine our previous analysis [16].

Closely related to the chiral expansion of the baryon masses is the pion–nucleon σ –term [3, 7, 21] (and also the two kaon–nucleon σ –terms, $\sigma_{KN}^{(1,2)}$). These σ –terms measure the

^{#4}Notice that only recently a systematic effective field theory formulation for spin-3/2 fields has become available for the two-flavor case [15].

strengths of the scalar quark condensates in the proton and vanish in the chiral limit of zero quark masses. They are thus particularly suited to test our understanding of the mechanism of the spontaneous and explicit chiral symmetry breaking. Again, at present no clear picture concerning the quark mass expansion of these quantities has emerged, although it is believed that the dispersion-theoretical analysis of Gasser, Leutwyler and Sainio [22] has significantly sharpened our understanding of relating $\sigma_{\pi N}$ as extracted from the pion-nucleon scattering data to the expectation value of $\hat{m}(\bar{u}u + \bar{d}d)$ in the proton, where $\hat{m} = (m_u + m_d)/2$ denotes the average light quark mass. Of particular interest is the so-called strangeness content of the proton, i.e. the matrix element $\langle p | m_s \bar{s}s | p \rangle$. It can be indirectly inferred from the analysis of the pion-nucleon σ -term. The present status can be summarized by the statement that this matrix element is non-vanishing but also not particularly large, as one deduces from the lowest order quark mass analysis [3] (for a status report, see e.g. Ref.[23]). Our aim is to show to what accuracy one can at present make statements about the quark mass expansion of the σ -terms. For some comprehensive reviews, see e.g. [24, 25, 26, 27].

The paper is organized as follows. In section 2 we write down the effective meson-baryon Lagrangian, i.e. all terms which contribute to the baryon masses and the σ -terms. The minimal number of contact terms up-to-and-including $\mathcal{O}(q^4)$ consists of seven terms of dimension two and seven terms of dimension four. These terms are accompanied by so-called low-energy constants (LECs), i.e. coupling constants not fixed by chiral symmetry. We remark that some of the dimension four LECs can be absorbed in some of the dimension two ones since they amount to quark mass renormalizations of the latter. This ambiguity is also discussed. In section 3 we give the formulae for the fourth order contribution to the octet baryon masses and the pion-nucleon σ -term. We also discuss the renormalization to render the divergent loop diagrams at order q^4 finite. This is in marked contrast to the calculation of the leading non-analytic corrections which are all finite. Section 4 deals with the estimate of the fourteen LECs from resonance exchange. We consider contributions from the spin-3/2 decuplet, the spin-1/2 octet of even-parity excited baryon resonances and from t-channel scalar meson exchange. Considering loop graphs with intermediate resonances leads to divergences to leading order. The corresponding renormalization constants can not be fixed from resonance parameters. This problem was incorrectly treated in [16]. However, using the baryon masses and the pion-nucleon σ -term, we are able to fix all but one of these coefficients. We show that the corresponding LECs and physical observables are essentially insensitive to the choice of this parameter which is bounded phenomenologically. We also address the question of two (and higher) loop contributions to the LECs. This is mandated by the fact that the estimation of the LECs for the baryon masses and σ -terms involves Goldstone boson loops (treated in a particular fashion as explained in section 4) and intermediate resonance propagators. Therefore, a non-trivial reordering of the chiral expansion for such graphs is expected. As we will show two-loop graphs only contribute to some of the subleading LECs at the same order as the one-loop diagrams. Most of the two-loop contributions can be completely absorbed in a redefinition of the one loop renormalization parameters. The results are given in section 5 including a detailed study of the dependence on certain

input parameters. A short summary and outlook is given in section 6. Some lengthy formulae and technicalities are relegated to the appendices.

2 Effective Lagrangian

To perform the calculations, we make use of the effective meson–baryon Lagrangian. To construct the various terms, we start from the relativistic formulation which is then reduced to the heavy fermion limit. This has the advantage of automatically generating all kinematical $1/m$ corrections with the correct coefficients, see e.g. appendix A of Ref.[28]. However, we do not spell out the relativistic Lagrangian but directly the heavy mass form emerging from it. In the extreme non–relativistic limit [14, 28], the baryons are characterized by a four–velocity v_μ . In this approach, there is a one–to–one correspondence between the expansion in small momenta and quark masses and the expansion in Goldstone boson loops, i.e. a consistent power counting scheme emerges. Our notation is identical to the one used in [7] and we discuss here only the new terms necessary for the calculations of the masses and σ –terms. The pseudoscalar Goldstone fields ($\phi = \pi, K, \eta$) are collected in the 3×3 unimodular, unitary matrix $U(x)$,

$$U(\phi) = u^2(\phi) = \exp\{i\phi/F_\pi\} \quad (1)$$

with F_π the pseudoscalar decay constant (in the chiral limit), and

$$\phi = \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} . \quad (2)$$

Under $SU(3)_L \times SU(3)_R$, $U(x)$ transforms as $U \rightarrow U' = LUR^\dagger$, with $L, R \in SU(3)_{L,R}$. The matrix B denotes the baryon octet,

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} . \quad (3)$$

Under $SU(3)_L \times SU(3)_R$, B transforms as any matter field,

$$B \rightarrow B' = K B K^\dagger , \quad (4)$$

with $K(U, L, R)$ the compensator field representing an element of the conserved subgroup $SU(3)_V$. The effective Lagrangian takes the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\phi B}^{(1)} + \mathcal{L}_{\phi B}^{(2)} + \mathcal{L}_{\phi B}^{(3)} + \mathcal{L}_{\phi B}^{(4)} + \mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(4)} \quad (5)$$

where the chiral dimension (i) counts the number of derivatives and/or meson mass insertions. Before discussing the various terms in \mathcal{L}_{eff} , a few words about the power counting

are in order. Clearly, derivatives (momenta) and meson masses count as order q . For the quark masses, we stick to the standard scenario, which prescribes a power q^2 to scalar and pseudoscalar external sources, in particular to the eigenvalues of the quark mass matrix \mathcal{M} . It would certainly be interesting to study the alternative approach in which the quark masses are counted as order q only. Clearly, one then would have to deal with much more terms at a given fixed order. We do not attempt such a 'generalized' analysis here. We remark that the dimension three operators in $\mathcal{L}_{\phi B}^{(3)}$ do not contribute directly to the masses but are needed for the renormalization of the baryon self-energy.

The form of the lowest order meson-baryon Lagrangian is

$$\mathcal{L}_{\phi B}^{(1)} = i \bar{B} v \cdot D B + F \text{Tr}(\bar{B} S_\mu [u^\mu, B]) + D \text{Tr}(\bar{B} S_\mu \{u^\mu, B\}) , \quad (6)$$

where $D \simeq 3/4$ and $F \simeq 1/2$ are the two axial-vector coupling constants, $D_\mu = \partial_\mu + \Gamma_\mu$ is the chiral covariant derivative^{#5} and S_μ denotes the covariant spin-operator à la Pauli-Lubanski. The meson Lagrangian $\mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(4)}$ is given in [19].

The dimension two meson-baryon Lagrangian can be written as (we only enumerate the terms which contribute)

$$\mathcal{L}_{\phi B}^{(2)} = \mathcal{L}_{\phi B}^{(2,\text{br})} + \sum_{i=1}^4 b_i O_i^{(2)} + \mathcal{L}_{\phi B}^{(2,\text{rc})} , \quad (7)$$

with the $O_i^{(2)}$ monomials in the fields of chiral dimension two discussed below. The explicit symmetry breaking terms are collected in $\mathcal{L}_{\phi B}^{(2,\text{br})}$ [7],

$$\mathcal{L}_{\phi B}^{(2,\text{br})} = b_D \text{Tr}(\bar{B} \{\chi_+, B\}) + b_F \text{Tr}(\bar{B} [\chi_+, B]) + b_0 \text{Tr}(\bar{B} B) \text{Tr}(\chi_+) , \quad (8)$$

i.e. it contains three low-energy constants $b_{0,D,F}$. $\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u$ is proportional to the quark mass matrix $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ since $\chi = 2B\mathcal{M}$. Here, $B = -\langle 0 | \bar{q}q | 0 \rangle / F_\pi^2$ is the order parameter of the spontaneous symmetry violation. We assume $B \gg F_\pi$. In what follows, we will work to order q^4 and thus have to include terms with derivatives on the Goldstone boson fields. The minimal set of such terms is given by

$$\begin{aligned} \sum_{i=1,2,3,8} b_i O_i^{(2)} &= b_1 \text{Tr}(\bar{B} [u_\mu, [u^\mu, B]]) + b_2 \text{Tr}(\bar{B} [u_\mu, \{u^\mu, B\}]) \\ &+ b_3 \text{Tr}(\bar{B} \{u_\mu, \{u^\mu, B\}\}) + b_8 \text{Tr}(\bar{B} B) \text{Tr}(u^\mu u_\mu) \end{aligned} \quad (9)$$

with $u_\mu = iu^\dagger \partial_\mu U u^\dagger$. A couple of remarks are in order. First, there are also terms of the type $\bar{B}(v \cdot u)^2 B$. Because of the identity (in d space-time dimensions)

$$\int d^d k \frac{(v \cdot k)^2}{k^2 - M_\pi^2} = \frac{1}{d} \int d^d k \frac{k^2}{k^2 - M_\pi^2} , \quad (10)$$

^{#5}For the calculations performed here, we only need the partial derivative in D_μ .

which holds for all the graphs to be considered later on, the respective coupling constants can completely be absorbed in the $b_{1,2,3,8}$. Second, we heavily make use of the Cayley–Hamilton identity for any traceless 3×3 matrix X ,

$$\text{Tr}(\bar{B}\{X^2, B\}) + \text{Tr}(\bar{B}XB X) - \frac{1}{2}\text{Tr}(\bar{B}B)\text{Tr}(X^2) - \text{Tr}(\bar{B}X)\text{Tr}(BX) = 0 \quad . \quad (11)$$

This allows e.g. to eliminate the term of the type $\text{Tr}(\bar{B}u_\mu)\text{Tr}(u^\mu B)$ in terms of the O_1 , O_3 and O_8 , i.e.

$$\begin{aligned} \frac{1}{4}\text{Tr}(\bar{B}[u_\mu, [u^\mu, B]]) + \frac{3}{4}\text{Tr}(\bar{B}\{u_\mu, \{u^\mu, B\}\}) \\ - \frac{1}{2}\text{Tr}(\bar{B}B)\text{Tr}(u_\mu u^\mu) - \text{Tr}(\bar{B}u_\mu)\text{Tr}(u^\mu B) = 0 \quad . \end{aligned} \quad (12)$$

The basis used in Ref.[16] was overcomplete and thus a larger number of terms was counted. This does, however, not directly matter for the calculations.^{#6} There are also relativistic corrections of dimension two, i.e. $1/m$ corrections with fixed coefficients. These read

$$\begin{aligned} \mathcal{L}_{\phi B}^{(2,\text{rc})} = & -\frac{iD}{2\overset{\circ}{m}}\text{Tr}(\bar{B}S_\mu[D^\mu, \{v \cdot u, B\}]) - \frac{iF}{2\overset{\circ}{m}}\text{Tr}(\bar{B}S_\mu[D^\mu, [v \cdot u, B]]) \\ & -\frac{iF}{2\overset{\circ}{m}}\text{Tr}(\bar{B}S_\mu[v \cdot u, [D^\mu, B]]) - \frac{iD}{2\overset{\circ}{m}}\text{Tr}(\bar{B}S_\mu\{v \cdot u, [D^\mu, B]\}) \\ & + \frac{D^2 - 3F^2}{24\overset{\circ}{m}}\text{Tr}(\bar{B}[v \cdot u, [v \cdot u, B]]) - \frac{D^2}{12\overset{\circ}{m}}\text{Tr}(\bar{B}B)\text{Tr}(v \cdot u v \cdot u) \\ & - \frac{1}{2\overset{\circ}{m}}\text{Tr}(\bar{B}[D_\mu, [D^\mu, B]]) + \frac{1}{2\overset{\circ}{m}}\text{Tr}(\bar{B}[v \cdot D, [v \cdot D, B]]) \\ & - \frac{DF}{4\overset{\circ}{m}}\text{Tr}(\bar{B}[v \cdot u, \{v \cdot u, B\}]) \quad , \end{aligned} \quad (13)$$

where $\overset{\circ}{m}$ denotes the average baryon octet mass in the chiral limit. All low-energy constants in $\mathcal{L}_{\phi B}^{(2)}$ are finite. The splitting of the dimension two meson–baryon Lagrangian in Eq.(7) is motivated by the fact that while the first three terms appear in tree and loop graphs, the latter eleven only come in via loops. We remark that the LECs b_i ($i = 0, 1, 2, 3, 8, D, F$) have dimension mass^{-1} . Notice also that the last three terms in Eq.(13) could be absorbed in the LECs $b_{1,2,3}$. For our later resonance saturation estimates, we prefer to keep them separately.

There are seven independent terms contributing at dimension four, which can be deduced from the over-complete basis

$$\mathcal{L}_{\phi B}^{(4)} = \sum_{i=1}^8 d_i O_i^{(4)}$$

^{#6}Notice that for facilitating the comparison with the results of Ref.[16], we keep the numbering of the terms used there.

$$\begin{aligned}
&= d_1 \text{Tr}(\bar{B}[\chi_+, [\chi_+, B]]) + d_2 \text{Tr}(\bar{B}[\chi_+, \{\chi_+, B\}]) \\
&+ d_3 \text{Tr}(\bar{B}\{\chi_+, \{\chi_+, B\}\}) + d_4 \text{Tr}(\bar{B}\chi_+) \text{Tr}(\chi_+ B) \\
&+ d_5 \text{Tr}(\bar{B}[\chi_+, B]) \text{Tr}(\chi_+) + d_6 \text{Tr}(\bar{B}\{\chi_+, B\}) \text{Tr}(\chi_+) \\
&+ d_7 \text{Tr}(\bar{B}B) \text{Tr}(\chi_+) \text{Tr}(\chi_+) + d_8 \text{Tr}(\bar{B}B) \text{Tr}(\chi_+^2) .
\end{aligned} \tag{14}$$

We choose to eliminate the d_6 -term by use of the Cayley–Hamilton identity Eq.(11) for $X = \chi_+ - \text{Tr}(\chi_+)/3$. The d_i have dimension mass^{-3} . It is important to note that some of the d_i simply amount to quark mass renormalizations of some of the dimension two LECs [8]. To be specific, one can absorb the effects of d_5 and d_7 in b_F and b_0 , respectively, as follows

$$b_F \rightarrow b_F - d_5 \text{Tr}(\chi_+) , \quad b_0 \rightarrow b_0 - d_7 \text{Tr}(\chi_+) . \tag{15}$$

This is a very general phenomenon of CHPT calculations in higher orders. For example, in $\pi\pi$ scattering there are six LECs at two loop order (q^6) [29], but only two new independent terms $\sim s^3$ and $\sim s M_\pi^4$. The other four LECs make the q^4 counter terms $\bar{\ell}_i$ ($i = 1, 2, 3, 4$) quark mass dependent. At this point, one has two options. One can either treat the higher order LECs as independent from the lower order ones or lump them together to minimize the number of independent terms. In the latter case, one needs to refit the numerical values of the lower dimension LECs. We prefer to work with the first option and treat all the d_i separately from the b_i . Therefore, we have fourteen LECs not fixed by chiral symmetry in addition to the F and D coupling constants (subject to the constraint $F + D = g_A = 1.25$) from the lowest order Lagrangian $\mathcal{L}_{\phi B}^{(1)}$. What we have to calculate are all one-loop graphs with insertions from $\mathcal{L}_{\phi B}^{(1,2)}$ and tree graphs from $\mathcal{L}_{\phi B}^{(2,4)}$. We stress that we do not include the spin-3/2 decuplet in the effective field theory [6], but rather use these fields to estimate the pertinent low-energy constants (resonance saturation principle). We therefore strictly expand in small quark masses and external momenta (collectively denoted by ' q ') with no recourse to large N_c arguments.

3 Baryon masses, πN σ -term and renormalization

In this section, we assemble the formulae for the quark mass squared contribution to the baryon masses and the σ -term. We also discuss the necessary renormalization of the LECs d_i since at order q^4 the meson loop contributions to the baryon masses and the σ -terms are no longer finite.

3.1 Fourth order contribution to the baryon masses

In general, the quark mass expansion of the octet baryon masses takes the form

$$m = \bar{m} + \sum_q B_q m_q + \sum_q C_q m_q^{3/2} + \sum_q D_q m_q^2 + \dots \tag{16}$$

modulo logs. The coefficients B_q, C_q, D_q are state-dependent. Furthermore, they include contributions proportional to the low-energy constants enumerated in the previous section. Our aim is to evaluate the terms of fourth order in the chiral counting. The lower

order terms $\sim m_q$ and $\sim m_q^{3/2}$ for the baryon masses are standard, we use here the same notation as Ref.[7]. Calculating the one loop graphs shown in Fig.1 and the counter terms $\sim d_i$ not shown in that figure, the q^4 contribution to any octet baryon mass m_B can be written as

$$m_B^{(4)} = \epsilon_{1,B}^P M_P^4 + \epsilon_{2,B}^{PQ} M_P^2 M_Q^2 + \epsilon_{3,B}^P \frac{M_P^4}{\Lambda_\chi^2} \ln\left(\frac{M_P^2}{\lambda^2}\right) + \epsilon_{4,B}^{PQ} \frac{M_P^2 M_Q^2}{\Lambda_\chi^2} \ln\left(\frac{M_P^2}{\lambda^2}\right) , \quad (17)$$

with $P, Q = \pi, K, \eta$ and λ the scale of dimensional regularization. Also, $\Lambda_\chi = 4\pi F_\pi$ is related to the scale of chiral symmetry breaking. In fact, this is the canonical prefactor appearing in one loop integrals as performed here. We remark that the graphs with an insertion from $\mathcal{L}_{\phi B}^{(2)}$ on the baryon propagator lead to a state-dependent shift, i.e. they contribute to the baryon mass splittings at order q^4 . The exception to this rule is the term proportional to b_0 . Expanded to quadratic order in the meson fields, it takes the form

$$4 B b_0 \text{Tr}(\bar{B}B) \text{Tr}(\mathcal{M}) - \frac{2}{F^2} B b_0 \text{Tr}(\bar{B}B) \text{Tr}(\phi^2 \mathcal{M}) + \mathcal{O}(\phi^4) \quad (18)$$

and thus its contribution to the baryon mass can be completely absorbed in the octet mass in the chiral limit,

$$\overset{\circ}{m} \text{Tr}(\bar{B}B) - b_0 \text{Tr}(\bar{B}B) \text{Tr}(\chi_+) = \overset{\circ}{m}' \text{Tr}(\bar{B}B) . \quad (19)$$

Therefore, as in the case of the q^3 calculation [5, 7] one needs additional information (like from one of the σ -terms) to disentangle b_0 and $\overset{\circ}{m}$. The explicit form of the state-dependent prefactors for the nucleon, i.e. the $\epsilon_{i,N}^{P(Q)}$, is (for the LECs d_i the renormalized, finite values enter as denoted by the superscript 'r', see next paragraph, and we do not show their explicit scale dependence)

$$\begin{aligned} \epsilon_{1,N}^\pi &= -4(4d_1^r + 2d_5^r + d_7^r + 3d_8^r) , \quad \epsilon_{2,N}^{\pi K} = 8(4d_1^r - 2d_2^r - d_5^r - 2d_7^r + 2d_8^r) , \\ \epsilon_{1,N}^K &= 16(-d_1^r + d_2^r - d_3^r + d_5^r - d_7^r - d_8^r) , \\ \epsilon_{3,N}^\pi &= 3(b_D + b_F + 2b_0 - b_1 - b_2 - b_3 - 2b_8) - \frac{3(D+F)^2}{4 \overset{\circ}{m}} , \\ \epsilon_{3,N}^K &= 2(3b_D - b_F + 4b_0 - 3b_1 + b_2 - 3b_3 - 4b_8) - \frac{5D^2 - 6DF + 9F^2}{6 \overset{\circ}{m}} , \\ \epsilon_{3,N}^\eta &= 2b_0 - 3b_1 + b_2 - \frac{1}{3}b_3 - 2b_8 - \frac{(D-3F)^2}{12 \overset{\circ}{m}} , \quad \epsilon_{4,N}^{\pi\pi} = 0 , \quad \epsilon_{4,N}^{\pi K} = 0 , \\ \epsilon_{4,N}^{K\pi} &= (5D^2 - 6DF + 9F^2)(2b_F + b_0) - \frac{9}{2}(D-F)^2(2b_D + b_0) \\ &\quad - \frac{1}{6}(D+3F)^2(3b_0 - 2b_D) , \\ \epsilon_{4,N}^{KK} &= -2(5D^2 - 6DF + 9F^2)(-b_D + b_F - b_0) - 9(D-F)^2 b_0 \\ &\quad - \frac{1}{3}(D+3F)^2(3b_0 + 4b_D) , \\ \epsilon_{4,N}^{\eta\pi} &= -b_D + \frac{5}{3}b_F , \quad \epsilon_{4,N}^{\eta K} = \frac{8}{3}b_D - \frac{8}{3}b_F . \end{aligned} \quad (20)$$

The corresponding coefficients for the Λ , Σ and the Ξ are collected in Appendix A. A few remarks are in order. First, we have $\epsilon_{1,B}^\eta = \epsilon_{2,B}^{P\eta} = \epsilon_{4,B}^{P\eta} = 0$ making use of the Gell-Mann–Okubo relation for the pseudoscalar meson masses. In these terms, the deviations from the GMO limit enters only at order q^6 . Second, the $\epsilon_{4,B}^{PP}$ could be absorbed in the $\epsilon_{3,B}^P$. We prefer to keep the more symmetric notation given in Eq.(20). Third, we stress that the mixed terms $\sim M_P^2 M_Q^2$ were not considered in most of the existing investigations. Notice that we have kept the baryon mass in the chiral limit. In the fourth order terms, it could simply be substituted by the physical mass (the difference being of higher order).

3.2 Renormalization

The fourth order contribution to the baryon masses are no longer finite. The appearance of these divergences is in marked contrast with the q^3 calculation which is completely finite (in the heavy fermion approach). To be precise, we calculate the baryon self-energy $\Sigma_B(\omega)$, which is related to the baryon propagator $S_B(\omega)$ via

$$S_B(\omega) = \frac{i}{p \cdot v - \overset{\circ}{m} - \Sigma_B(\omega)} , \quad (21)$$

with $p_\mu = \overset{\circ}{m} v_\mu + l_\mu$, $v \cdot l \ll \overset{\circ}{m}$ and $\omega = v \cdot l$. The propagator develops a pole at $p = m_B v$, with m_B the renormalized (physical) baryon mass,

$$m_B = \overset{\circ}{m} + \Sigma_B(0) . \quad (22)$$

The nucleon wave-function renormalization is determined by the residue of the propagator at the physical mass,

$$S_B(\omega) = \frac{i Z_B}{p \cdot v - m_B} , \quad Z_B = 1 + \Sigma'_B(0) . \quad (23)$$

To calculate the pertinent loop graphs, we use dimensional regularization and all appearing divergences can be absorbed in the LECs d_i

$$d_i = d_i^r(\lambda) + \frac{\Gamma_i}{F_\pi^2} L . \quad (24)$$

with λ the scale of dimensional regularization and

$$L = \frac{\lambda^{d-4}}{16\pi^2} \left\{ \frac{1}{d-4} - \frac{1}{2} [\ln(4\pi) + 1 - \gamma_E] \right\} . \quad (25)$$

Here, $\gamma_E = 0.5772215$ is the Euler-Mascheroni constant. The scale dependence of the $d_i^r(\lambda)$ follows from Eq.(24):

$$d_i^r(\lambda_2) = d_i^r(\lambda_1) + \frac{\Gamma_i}{(4\pi F_\pi)^2} \ln \frac{\lambda_1}{\lambda_2} . \quad (26)$$

In what follows, we set $\lambda = 1$ GeV. The corresponding Γ_i read

$$\begin{aligned}
\Gamma_1 &= -\frac{1}{6}b_1 + \frac{1}{18}b_3 + \frac{1}{36}(7 + 15D^2 + 27F^2)b_D \\
&\quad + \frac{1}{72\overset{\circ}{m}}(D^2 - 3F^2) - \frac{3}{8}\left(-\frac{1}{2}(D^2 - 3F^2)b_0 + \frac{2}{3}DFb_F\right) \\
\Gamma_2 &= \frac{1}{4}b_2 + \frac{1}{12}(1 + 24D^2)b_F + \frac{13}{96\overset{\circ}{m}}DF \\
\Gamma_3 &= -\frac{3}{4}b_1 - \frac{1}{12}b_3 + \frac{1}{4}(2 + 3(D^2 - 3F^2))b_D \\
&\quad + \frac{7}{128\overset{\circ}{m}}(D^2 - 3F^2) - \frac{3}{4}\left(-\frac{3}{4}(D^2 - 3F^2)b_0 - 3DFb_F\right) \\
\Gamma_4 &= \frac{3}{2}b_1 - \frac{1}{18}b_3 + \frac{1}{2}\left(-\frac{11}{9} - \frac{9}{4}\left(\frac{68}{27}D^2 - 4F^2\right)\right)b_D \\
&\quad - \frac{1}{8\overset{\circ}{m}}(D^2 - 3F^2) - \frac{9}{4}\left(\frac{1}{3}D^2 - F^2\right)b_0 \\
\Gamma_5 &= -\frac{13}{18}b_2 + \frac{1}{2}\left(\frac{11}{9} - \frac{13}{3}D^2 - 3F^2\right)b_F - \frac{5}{2}DFb_0 - \frac{13}{36\overset{\circ}{m}}DF \\
\Gamma_7 &= -\frac{1}{4}b_1 - \frac{35}{108}b_3 - \frac{11}{18}b_8 - \frac{7}{128\overset{\circ}{m}}\left(\frac{35}{27}D^2 + F^2\right) \\
&\quad - \frac{1}{8}\left(-\frac{22}{9} + \frac{34}{3}D^2 + 6F^2\right)b_D + \frac{1}{8}\left(\frac{44}{9} - \frac{43}{3}D^2 - 21F^2\right)b_0 \\
\Gamma_8 &= -\frac{1}{4}b_1 - \frac{17}{36}b_3 - \frac{5}{6}b_8 - \frac{7}{128\overset{\circ}{m}}\left(\frac{17}{9}D^2 + F^2\right) - 3DFb_F \\
&\quad - \frac{1}{8}\left(-\frac{14}{9} + \frac{2}{3}D^2 + 6F^2\right)b_D + \frac{1}{2}\left(\frac{5}{3} - \frac{3}{4}(D^2 - 3F^2)\right)b_0
\end{aligned} \tag{27}$$

We note that for performing this renormalization, one has to include the terms

$$\begin{aligned}
&-2i\left(\frac{10}{3}D^2 + F^2\right)\text{Tr}(\bar{B}[v \cdot D, [v \cdot D, [v \cdot D, B]]]) \\
&+ i\frac{3}{4}(D^2 - 3F^2)\text{Tr}(\bar{B}\{\chi_+, [v \cdot D, B]\}) - i\frac{5}{2}DF\text{Tr}(\bar{B}[\chi_+, [v \cdot D, B]]) \\
&- i\frac{3}{2}\left(\frac{13}{9}D^2 + F^2\right)\text{Tr}(\bar{B}[v \cdot D, B])\text{Tr}(\chi_+) \\
&- \frac{3}{8\overset{\circ}{m}}(D^2 - 3F^2)\text{Tr}(\bar{B}\{\chi_+, [D^\mu, [D_\mu, B]]\}) \\
&+ \frac{5}{4\overset{\circ}{m}}DF\text{Tr}(\bar{B}[\chi_+, [D^\mu, [D_\mu, B]]]) \\
&+ \frac{3}{4}\left(\frac{13}{9}D^2 + F^2\right)\text{Tr}(\bar{B}[D^\mu, [D_\mu, B]])\text{Tr}(\chi_+) \\
&+ \left\{\frac{3}{\overset{\circ}{m}}(D^2 - 3F^2) - 3\left(\frac{1}{3}D^2 b_0 - b_D(D^2 - 3F^2) - 6DFb_F\right)\right\} \\
&\quad \times \text{Tr}(\bar{B}\{\chi_+, [v \cdot D, [v \cdot D, B]]\})
\end{aligned}$$

$$\begin{aligned}
& + \left\{ -\frac{10}{\hat{m}} DF - 4 \left(b_F \left(5D^2 + 9F^2 \right) - 10DF b_D \right) \right\} \text{Tr}(\bar{B}[\chi_+, [v \cdot D, [v \cdot D, B]]]) \\
& + \left\{ -\frac{6}{\hat{m}} \left(\frac{13}{9} D^2 + F^2 \right) - 6 \left(b_0 \left(\frac{8}{3} D^2 + 6F^2 \right) + b_D \left(2F^2 + \frac{26}{9} D^2 \right) + 4DF b_F \right) \right\} \\
& \quad \times \text{Tr}(\bar{B}[v \cdot D, [v \cdot D, B]]) \text{Tr}(\chi_+) \\
& + \frac{1}{\hat{m}} (10D^2 + 18F^2) \text{Tr}(\bar{B}[v \cdot D, [v \cdot D, [D^\mu, [D_\mu, B]]]]) \\
& - \frac{10}{\hat{m}} \left(\frac{10}{3} D^2 + 6F^2 \right) \text{Tr}(\bar{B}[v \cdot D, [v \cdot D, [v \cdot D, [v \cdot D, B]]]])
\end{aligned} \tag{28}$$

in the effective Lagrangian (an overall factor LF_π^{-2} has been scaled out and the finite pieces related to these operators can be neglected). These, however, do not directly contribute to the masses and σ -terms and are therefore not listed in section 3.1. It is obvious that the dimension two terms also enter the renormalization procedure and thus the finite constants b_i appear in the Γ_i . This is a general feature of any renormalization beyond $\mathcal{O}(q^3)$ in heavy baryon CHPT.

3.3 Pion/kaon–nucleon σ -terms and strangeness content

Further information on the scalar sector of baryon CHPT is given by the scalar form factors or σ -terms which measure the strength of the various matrix elements $m_q \bar{q}q$ ($q = u, d, s$) in the proton. In a mass-independent renormalization scheme, one can define the following renormalization-group invariant quantities:

$$\begin{aligned}
\sigma_{\pi N}(t) &= \hat{m} \langle p' | \bar{u}u + \bar{d}d | p \rangle , \\
\sigma_{KN}^{(1)}(t) &= \frac{1}{2}(\hat{m} + m_s) \langle p' | \bar{u}u + \bar{s}s | p \rangle , \\
\sigma_{KN}^{(2)}(t) &= \frac{1}{2}(\hat{m} + m_s) \langle p' | -\bar{u}u + 2\bar{d}d + \bar{s}s | p \rangle ,
\end{aligned} \tag{29}$$

with $t = (p' - p)^2$ the invariant momentum transfer squared. The explicit form of the t -dependent σ -terms is rather lengthy and not very instructive. Here, we just discuss some general aspects and refer the reader to Ref.[30] for details. The most striking new feature compared to the $\mathcal{O}(q^3)$ analysis [7, 28, 36] is the appearance of t -dependent divergences. To be more specific, consider a typical diagram as shown in Fig.2. It is most convenient to calculate these diagrams in the Breit-frame [28]. For the case at hand, we get

$$\begin{aligned}
I_\sigma(t) &= \int \frac{d^4k}{(2\pi)^4} \frac{i^2 k(k+l)}{(k^2 - M_a^2)((k+l)^2 - M_a^2)} = i \left\{ (4M_a^2 - t) L \right. \\
& \left. - \frac{1}{16\pi^2} \left(M_a^2 - \frac{t}{2} - \frac{1}{2}(4M_a^2 - t) \ln \frac{M_a^2}{\lambda^2} - (2M_a^2 - t) \frac{\sqrt{4M_a^2 - t}}{\sqrt{t}} \arcsin\left(\frac{\sqrt{t}}{2M_a}\right) \right) \right\}
\end{aligned} \tag{30}$$

where we have suppressed the pertinent Clebsch–Gordan coefficient and M_a stands for a Goldstone boson mass in the SU(3) flavor basis. The renormalization of this diagram and

the other contributing to the σ -terms is somewhat tricky and is described in appendix B. Note that when combining all the terms from the various Feynman graphs (after renormalization), one is indeed left with terms proportional to $\text{const} \cdot t$. In fact, these t -dependent coefficients can not easily be estimated. Also, they are different for $\sigma_{\pi N}(t)$ and $\sigma_{KN}^{(1,2)}(t)$, respectively. The empirical information on $\sigma_{\pi N}(2M_\pi^2) - \sigma_{\pi N}(0) = 15 \text{ MeV}$ [22] is thus not sufficient to make predictions for the two KN σ -terms shifts, $\sigma_{KN}^{(1,2)}(2M_K^2) - \sigma_{KN}^{(1,2)}(0)$. We will therefore only give some rough estimates by simply parametrizing these t -dependent pieces in terms of coefficients which we can estimate to be of order one. Note that to order q^3 , the shifts from $t = 0$ to the respective Chang–Dashen points are finite and free of any LEC. In view of this, the nice result found in Ref.[7] for $\Delta\sigma_{\pi N} \simeq 15 \text{ MeV}$ by including the $\Delta(1232)$ explicitly as intermediate state must be considered accidental.

At $t = 0$, the corresponding formulae simplify considerably. The fourth order contribution to $\sigma_{\pi N}(0)$ takes the form

$$\sigma_{\pi N}^{(4)}(0) = M_\pi^2 \left[\epsilon_{1,\sigma}^P M_P^2 + \epsilon_{2,\sigma}^P \frac{M_P^2}{\Lambda_\chi^2} \ln\left(\frac{M_P^2}{\lambda^2}\right) + \epsilon_{3,\sigma}^{PQ} \frac{M_P^2}{\Lambda_\chi^2} \ln\left(\frac{M_Q^2}{\lambda^2}\right) \right], \quad (31)$$

with the $\epsilon_{i,\sigma}^{P(Q)}$ are related to the $\epsilon_{i,N}^{P(Q)}$, Eq.(20), and they are collected in Appendix C. These relations are, of course, a consequence of the Feynman–Hellmann theorem which states that

$$\sigma_{\pi N}(0) = \hat{m} \frac{\partial m_N}{\partial \hat{m}}. \quad (32)$$

The derivatives with respect to the quark masses can be converted into derivatives with respect to the Goldstone boson masses squared, for example

$$\hat{m} \frac{\partial}{\partial \hat{m}} M_K^4 \ln \frac{M_K^2}{\lambda^2} = M_\pi^2 M_K^2 \left\{ \frac{1}{2} + \ln \frac{M_K^2}{\lambda^2} \right\}. \quad (33)$$

Eq.(32) can therefore be evaluated easily. By a similar reasoning, one can deduce the strangeness content of the proton, i.e the strength of the matrix element $\langle p | m_s \bar{s}s | p \rangle$,

$$\langle p | m_s \bar{s}s | p \rangle = m_s \frac{\partial m_N}{\partial m_s}, \quad (34)$$

and the strangeness fraction y ,

$$y = \frac{2 \langle p | \bar{s}s | p \rangle}{\langle p | \bar{u}u + \bar{d}d | p \rangle} = \frac{M_\pi^2}{\sigma_{\pi N}(0)} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right)^{-1} m_s \frac{\partial m_N}{\partial m_s}. \quad (35)$$

The fourth order contribution to y , denoted by $y^{(4)}$, can be decomposed in the standard fashion (for the lower orders, see [7])

$$y^{(4)} = \frac{2 M_\pi^2}{\sigma_{\pi N}(0)} \left[\epsilon_{1,y}^P M_P^2 + \epsilon_{2,y}^P \frac{M_P^2}{\Lambda_\chi^2} \ln\left(\frac{M_P^2}{\lambda^2}\right) + \epsilon_{3,y}^{PQ} \frac{M_P^2}{\Lambda_\chi^2} \ln\left(\frac{M_Q^2}{\lambda^2}\right) \right], \quad (36)$$

and the $\epsilon_{i,y}^{P(Q)}$ are again related to the $\epsilon_{i,N}^{P(Q)}$, see appendix C. The more lengthy expressions for the two KN σ -terms can be found in Ref.[30]. This completes the formalism to study the scalar sector of baryon CHPT to fourth order in the meson masses.

4 Low-energy constants from resonance saturation

Clearly, we are not able to fix all the low-energy constants appearing in $\mathcal{L}_{\phi B}^{(2,4)}$ from data, even if we would resort to large N_c arguments. We will therefore use the principle of resonance saturation to estimate these constants. This works very accurately in the meson sector [31, 32, 33]. In the baryon case, one has to account for excitations of meson (R) and baryon (N^*) resonances. One writes down the effective Lagrangian with these resonances chirally coupled to the Goldstone bosons and the baryon octet, calculates the Feynman diagrams pertinent to the process under consideration and, finally, lets the resonance masses go to infinity (with fixed ratios of coupling constants to masses). This generates higher order terms in the effective meson-baryon Lagrangian with coefficients expressed in terms of a few known resonance parameters. Symbolically, we can write

$$\tilde{\mathcal{L}}_{\text{eff}}[U, B, R, N^*] \rightarrow \mathcal{L}_{\text{eff}}[U, B] . \quad (37)$$

It is important to stress that only after integrating out the heavy degrees of freedom from the effective field theory, one is allowed to perform the heavy mass limit for the ground-state baryon octet. Here, there are two relevant types of contributions. One comes from the excitation of intermediate baryon resonances, in particular from the spin-3/2 decuplet states. Concerning the higher baryon resonances, only the parity-even spin-1/2 octet including the Roper $N^*(1440)$ plays some role. In addition, there is t -channel scalar and vector meson exchange. It was already shown in Ref.[16] that the vectors do not come in at order q^4 . It is important to stress that for the resonance contribution to the baryon masses, one has to involve Goldstone boson loops. This is different from the normal situation like e.g. in form factors or scattering processes. One could argue that scalar meson exchange alone should provide the necessary strength of the scalar-isoscalar operators $\sim \text{Tr}(\chi_+)$. However, taking the phenomenological bounds on the scalar masses and coupling constants, these contributions are too small to explain e.g. the strengths of the symmetry breakers $b_{0,D,F}$ which have been determined to order q^3 in [7]. To be specific, if one performs the calculation in SU(2) and demands that the LEC c_1 , which can be fixed from the πN σ -term, to be saturated by scalar meson exchange only, one would need a mass to coupling constant ratio of $M_s/g_s = 220$ MeV. Here, g_s refers to the SU(2) coupling $g_s \bar{\Psi}_N \Psi_N S$. Such a number can not be obtained from standard scalar meson masses and their couplings to the nucleon (baryons). In Ref.[7] it was, however, shown, that one-loop graph with intermediate decuplet states can effectively produce such couplings. However, treating the resonances relativistically leads to four major complications.

- First, terms arise which are non-analytic in the meson masses. Clearly, to avoid any double counting and to be consistent with the requirements from analyticity, one should only consider the analytic terms in the meson masses generated by such loop diagrams. In fact, standard tree level resonance excitation generates the dimension two operators $b_{1,\dots,8}$ in the conventional fashion. Calculating tadpole diagrams to order q^4 with these b_i generated from resonance exchange exactly produces the non-analytic terms $\sim M_\phi^4 \ln M_\phi^2$ which also arise from the one-loop

graphs with intermediate decuplet states. This is why we have to dismiss such non-analytic contributions in the Goldstone boson masses in our estimation of the scalar-isoscalar LECs from loop graphs. We remark that this kind of equivalence has been already discussed for the SU(2) LEC c_2 which enter the calculation of the nucleon electromagnetic polarizabilities to order q^4 , see [34] [35].

- Second, to the order we are working, even the remaining analytic pieces are divergent (this was incorrectly treated in ref.[16]). Therefore, we can only determine the analytic resonance contribution up to three renormalization constants, two related to the decuplet (β_Δ , δ_Δ) and one to the Roper-octet (β_R). We solve this problem in the following manner. In addition to these three constants, we have as further unknowns the scalar couplings F_S and D_S and the octet baryon mass in the chiral limit. Fitting the four octet baryon masses and $\sigma_{\pi N}(0)$ allows us to express all LECs and physical observables in terms of β_R solely. Fortunately, all quantities are very insensitive to the choice of this parameter which we can bound by some phenomenological argument.
- Third, since the baryon excitations are treated relativistically as explained above, there is no more strict power counting [36] and thus we must address the question of two-loop contributions from diagrams with baryon excitations. To leading order in the resonance mass expansion, they give rise to the same divergences as the one loop graphs and the corresponding corrections can be completely absorbed in a redefinition of the parameters appearing at one loop. We also show that there is an additional divergence at order M_ϕ^4 in the Roper-octet graphs which modifies the finite one-loop parameter δ_R . However, the contributions to the baryon masses and the σ -terms $\sim \delta_R$ are very small and thus one does not need to know this parameter accurately. Using dimensional analysis, the two-loop corrections to β_R are found to be modest. For details, see paragraph 4.5.
- Fourth, the renormalization of the σ -terms is somewhat complicated. The divergences of the scalar form factors are renormalized through terms of the form $\partial\mathcal{L}/\partial m_q$. In general, one has $\partial\mathcal{L}/\partial m_u \neq \partial\mathcal{L}/\partial m_d \neq \partial\mathcal{L}/\partial m_s$. If one calculates in the isospin limit $m_u = m_d$, one can not disentangle the derivatives with respect to m_u and m_d any more. For the general renormalization procedure, one has to work with $m_u \neq m_d$. Applying resonance saturation, one effectively determines the LECs of the Lagrangian but not the ones related to the operators $\bar{u}u$, $\bar{d}d$ and $\bar{s}s$. By use of the Feynman-Hellmann theorem, the LECs related to the operators $\bar{q}q$ can be fixed from the ones which appear in the calculation of the baryon masses. This general theorem is proven in ref.[30]. It means that here we are only able to give the full order q^4 contribution to the pion-nucleon σ -term at $t = 0$ and can only determine $\sigma_{KN}^{(1,2)}(0)$ up some renormalization constant (since we work in the isospin limit). We also point out that we have not yet been able to generalize these methods to the momentum-dependent parts of the scalar form factors since their integrals appear which we can not work out analytically (for details, see ref.[30]).

Let us now consider the one-loop contributions from the decuplet, the $\frac{1}{2}^+$ -octet and from the scalar mesons, in order.

4.1 Decuplet contribution to the low-energy constants

Consider first the decuplet contribution. We treat these field relativistically and only at the last stage let the mass become very large. The pertinent interaction Lagrangian between the spin-3/2 fields (denoted by Δ), the baryon octet and the Goldstone bosons reads

$$\mathcal{L}_{\Delta B\phi} = \frac{\mathcal{C}}{2} \left\{ \bar{\Delta}^{\mu,abc} \Theta_{\mu\nu}(Z) (u^\nu)_a^i B_b^j \epsilon_{cij} + \bar{B}_j^b (u^\nu)_i^a \Theta_{\nu\mu}(Z) \Delta_{abc}^\mu \epsilon^{cij} \right\} , \quad (38)$$

with $'a, b, \dots, j'$ are $\text{SU}(3)_f$ indices and the coupling constant $\mathcal{C} = 1.2 \dots 1.8$ can be determined from the decays $\Delta \rightarrow B\pi$. The Dirac matrix operator $\Theta_{\mu\nu}(Z)$ is given by

$$\Theta_{\mu\nu}(Z) = g_{\mu\nu} - \left(Z + \frac{1}{2} \right) \gamma_\mu \gamma_\nu . \quad (39)$$

For the off-shell parameter Z , we use $Z = -0.3$ from the determination of the Δ contribution to the πN scattering volume a_{33} [37]. That is also consistent with recent studies of $\Delta(1232)$ contributions to the nucleons' electromagnetic polarizabilities [38] and to threshold pion photo- and electroproduction [39]. For the processes to be discussed, we only need the expanded form of u_μ ,

$$(u_\mu)_a^i = -\frac{1}{F_\pi} \partial_\mu \phi_a^i + \mathcal{O}(\phi^2) . \quad (40)$$

The propagator of the spin-3/2 fields has the form

$$G_{\beta\delta}(p) = -i \frac{\not{p} + m_\Delta}{p^2 - m_\Delta^2} \left(g_{\beta\delta} - \frac{1}{3} \gamma_\beta \gamma_\delta - \frac{2p_\beta p_\delta}{3m_\Delta^2} + \frac{p_\beta \gamma_\delta - p_\delta \gamma_\beta}{3m_\Delta} \right) , \quad (41)$$

with $m_\Delta = 1.38$ GeV the average decuplet mass. We have to evaluate diagrams as shown in Fig. 3. With the labeling of the momenta as in that figure, this leads to

$$I_\Delta(p) = \frac{-i \mathcal{C}^2}{2F_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{(k-p)^\sigma \Theta_{\sigma\rho}(Z) G^{\rho\mu}(k) \Theta_{\mu\nu}(Z) (k-p)^\nu}{(k-p)^2 - M_a^2 + i\epsilon} . \quad (42)$$

where the pertinent Clebsch-Gordan coefficient has been omitted and M_a as defined after Eq.(31). This integral is evaluated on the mass-shell of the external baryons, i.e at $\not{p} = \not{\bar{m}}$, and split into various contributions according to the power of momenta in the numerator and the number of propagators. Each such term is then expanded in powers of Goldstone boson masses up-to-and-including $\mathcal{O}(M_a^4)$ (modulo logs). Only then the large mass limit of the decuplet is taken. This then gives the contribution to the various LECs b_i and d_i modulo some renormalization constants. To be specific, consider the scalar integral to

which all contributions arising in evaluating I_Δ can be reduced

$$\begin{aligned}
& i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[(k-p)^2 - M_a^2 + i\epsilon][k^2 - m_\Delta^2 + i\epsilon]} \\
&= \left\{ 2L + \frac{1}{16\pi^2} \left[-1 + \frac{m_\Delta^2}{\dot{m}^2} \ln\left(\frac{m_\Delta^2}{\lambda^2}\right) - \frac{1}{\dot{m}^2} (m_\Delta^2 - \dot{m}^2) \ln\left(\frac{m_\Delta^2 - \dot{m}^2}{\lambda^2}\right) \right. \right. \\
&+ \frac{1}{\dot{m}^2 (m_\Delta^2 - \dot{m}^2)} \left(\dot{m}^2 - m_\Delta^2 \ln\left(\frac{m_\Delta^2}{\lambda^2}\right) + (m_\Delta^2 + \dot{m}^2) \ln\left(\frac{m_\Delta^2 - \dot{m}^2}{\lambda^2}\right) \right) M_a^2 \\
&+ \frac{1}{2(m_\Delta^2 - \dot{m}^2)^3} \left(-\dot{m}^2 - 2m_\Delta^2 \ln\left(\frac{m_\Delta^2}{\lambda^2}\right) + 4m_\Delta^2 \ln\left(\frac{m_\Delta^2 - \dot{m}^2}{\lambda^2}\right) \right) M_a^4 \\
&\left. \left. - \frac{1}{m_\Delta^2 - \dot{m}^2} M_a^2 \ln\left(\frac{M_a^2}{\lambda^2}\right) - \frac{m_\Delta^2}{(m_\Delta^2 - \dot{m}^2)^3} M_a^4 \ln\left(\frac{M_a^2}{\lambda^2}\right) \right] \right\}. \tag{43}
\end{aligned}$$

In total, the $M_a^2 \ln M_a^2$ terms cancel in I_Δ . As explained before, the remaining non-analytic pieces $\sim M_a^4 \ln M_a^2$ appearing in these integrals should not be retained, see also below. We can now work out the complete integral $I_\Delta(\not{p} = \dot{m})$ and find at the scale $\lambda = m_\Delta$

$$I_\Delta^{\text{analytic}} = \frac{-i\mathcal{C}^2}{6F_\pi^2} \left\{ -(2Z+1)m_\Delta M_a^2 - (4Z^2 + 5Z - \frac{3}{2}) \frac{1}{m_\Delta} M_a^4 \right\} 2L + \dots \tag{44}$$

where the ellipsis stands for constant and divergent pieces which have to be taken care of by standard mass renormalization terms of the type $\delta m_\Delta \text{Tr}(\bar{\Delta}\Delta)$, see e.g. Refs.[7, 36], and for subleading terms in the $1/m_\Delta$ -expansion at orders M_a^2 and M_a^4 . If one were to use the subleading finite pieces to estimate the LECs, the resulting values would be much too small. One notices that in this relativistic treatment, the dimension two and four LECs are no longer finite (as in the heavy baryon approach) [36]. The structure of Eq.(44) shows that the first and second term in the curly brackets will contribute to the $b_{0,D,F}$, and the d_i , respectively, after renormalization

$$I_\Delta^{\text{analytic}} = \frac{-i\mathcal{C}^2}{2F_\pi^2} \frac{1}{16\pi^2} \left\{ -m_\Delta M_a^2 \alpha_\Delta^{(2)} + \frac{1}{m_\Delta} M_a^4 \alpha_\Delta^{(4)} \right\}, \tag{45}$$

where $\alpha_\Delta^{(2,4)}$ are the finite renormalization constants related to the divergences appearing at order M_a^2 and M_a^4 , respectively. The $b_{1,2,3,8}$ are calculated from standard pion-nucleon scattering tree graphs with intermediate decuplet states (for details, see ref.[30]). As an important check, feeding these back into the tadpole diagrams one finds non-analytic pieces which exactly agree with the ones dropped from the integral I_Δ . These have the generic form

$$I_\Delta^{\text{non-analytic}} = (2Z^2 + Z - 1) \frac{M_a^4}{m_\Delta} \ln \frac{M_a^2}{\lambda^2}. \tag{46}$$

This proves the consistency of our procedure. Evaluating now the pertinent Clebsch–Gordan coefficients and defining

$$\epsilon_\Delta = \frac{\mathcal{C}^2}{12m_\Delta}(2Z^2 + Z - 1) , \quad \beta_\Delta = \frac{\mathcal{C}^2}{128\pi^2 F_\pi^2} m_\Delta \alpha_\Delta^{(2)} , \quad \delta_\Delta = -\frac{\mathcal{C}^2}{256\pi^2 F_\pi^2} \frac{1}{m_\Delta} \alpha_\Delta^{(4)} , \quad (47)$$

we arrive at the decuplet contribution to the various LECs

$$\begin{aligned} b_0^\Delta &= \frac{7}{3} \beta_\Delta , \quad b_D^\Delta = -\beta_\Delta , \quad b_F^\Delta = \frac{5}{6} \beta_\Delta , \\ b_1^\Delta &= -\frac{7}{12} \epsilon_\Delta , \quad b_2^\Delta = \epsilon_\Delta , \quad b_3^\Delta = -\frac{3}{4} \epsilon_\Delta , \quad b_8^\Delta = \frac{3}{2} \epsilon_\Delta , \\ d_1^\Delta &= -\frac{1}{18} \delta_\Delta , \quad d_2^\Delta = -\frac{1}{4} \delta_\Delta , \quad d_3^\Delta = -\frac{1}{2} \delta_\Delta , \quad d_4^\Delta = \frac{5}{9} \delta_\Delta , \\ d_5^\Delta &= \frac{13}{36} \delta_\Delta , \quad d_7^\Delta = \frac{19}{144} \delta_\Delta , \quad d_8^\Delta = \frac{3}{4} \delta_\Delta , \end{aligned} \quad (48)$$

where β_Δ and δ_Δ have to be determined as described below.

4.2 $1/2^+$ –octet contribution to the low–energy constants

The next multiplet of excited states is the octet of even–parity Roper–like spin–1/2 fields. While it was argued in Ref.[6] that these play no role, a more recent study seems to indicate that one can not completely neglect contributions from these states to e.g. the decuplet magnetic moments [40]. It is thus important to investigate the possible contribution of these baryon resonances to the LECs. The octet consists of the $N^*(1440)$, the $\Sigma^*(1660)$, the $\Lambda^*(1600)$ and the Ξ^* . For the mass of the latter, we use the Gell-Mann–Okubo formula,

$$m_{\Xi^*} = \frac{3}{2} m_{\Lambda^*} + \frac{1}{2} m_{\Sigma^*} - m_{N^*} = 1.79 \text{ GeV} , \quad (49)$$

so that we find for the average Roper octet mass

$$m_R = 1.63 \text{ GeV} , \quad (50)$$

which is 490 MeV above the mean of the ground state octet. Notice that we denote the spin–1/2⁺ octet by ‘ R ’.^{#7} The effective Lagrangian of the Roper octet coupled to the ground state baryons takes the form

$$\begin{aligned} \mathcal{L}_{\text{eff}}(B, R) &= \mathcal{L}_0(R) + \mathcal{L}_{\text{int}}(B, R) , \\ \mathcal{L}_0(R) &= i \text{Tr}(\bar{R} \gamma^\mu [D_\mu, R]) - m_R \text{Tr}(\bar{R} R) , \\ \mathcal{L}_{\text{int}}(B, R) &= \frac{D_R}{4} \left[\text{Tr}(\bar{R} \gamma^\mu \gamma_5 \{u_\mu, B\}) + \text{h.c.} \right] \\ &\quad + \frac{F_R}{4} \left[\text{Tr}(\bar{R} \gamma^\mu \gamma_5 [u_\mu, B]) + \text{h.c.} \right] . \end{aligned} \quad (51)$$

^{#7}This should not be confused with the same symbol denoting any meson resonance.

The numerical values of the axial–vector coupling constants D_R and F_R are determined in appendix D (generalizing the results for the two–flavor case [41]). With these definitions, we evaluate the same type of graph as shown in Fig. 3 (dropping again the Clebsch–Gordan coefficients),

$$I_R(p) = - \int \frac{d^4 k}{(2\pi)^4} \frac{(\not{p} - \not{k})(\not{k} - m_R)(\not{p} - \not{k})}{[(p - k)^2 - M_a^2 + i\epsilon][k^2 - m_R^2 + i\epsilon]} . \quad (52)$$

which by inspection leads to the same kind of terms as in Eq.(42), i.e. with k^n ($n = 0, 1, 2, 3$) in the numerator combined with one or two propagators. Expanding these terms again first in powers of the Goldstone boson mass M_a and then in powers of $1/m_R$, one finds at the scale $\lambda = m_R$ that the terms of order M_a^2 diverge whereas the ones of order M_a^4 are finite. After appropriate renormalization, we have

$$I_R^{\text{analytic}}(\not{p} = \not{m}) = - \frac{i}{16\pi^2} \left\{ m_R M_a^2 \alpha_R^{(2)} + \frac{\overset{\circ}{m}}{2m_R^2} M_a^4 \right\} . \quad (53)$$

As before, the contributions to the $b_{1,2,3,8}$ are calculated from tree–level graphs with intermediate Roper states (for details, see ref.[30]). Evaluating now the pertinent Clebsch–Gordan coefficients and defining

$$\epsilon_R = - \frac{1}{8m_R} , \quad \beta_R = \frac{1}{256\pi^2 F_\pi^2} m_R \alpha_R^{(2)} , \quad \delta_R = \frac{1}{1024\pi^2 F_\pi^2} \frac{\overset{\circ}{m}}{m_R^2} , \quad (54)$$

we arrive at spin-1/2⁺ octet contribution to the various LECs

$$\begin{aligned} b_0^R &= \left(\frac{13}{9} D_R^2 + F_R^2 \right) \beta_R , \quad b_D^R = \frac{1}{2} (3F_R^2 - D_R^2) \beta_R , \quad b_F^R = \frac{5}{3} D_R F_R \beta_R , \\ b_1^R &= -(D_R^2 - 3F_R^2) \frac{\epsilon_R}{6} , \quad b_2^R = D_R F_R \epsilon_R , \quad b_3^R = 0 , \quad b_8^R = \frac{1}{3} D_R^2 \epsilon_R , \\ d_1^R &= -(D_R^2 - 3F_R^2) \frac{\delta_R}{18} , \quad d_2^R = -\frac{1}{2} D_R F_R \delta_R , \quad d_3^R = -(D_R^2 - 3F_R^2) \frac{\delta_R}{4} , \\ d_4^R &= (D_R^2 - 3F_R^2) \frac{\delta_R}{3} , \quad d_5^R = D_R F_R \frac{13\delta_R}{18} , \quad d_7^R = \left(\frac{35}{26} D_R^2 + F_R^2 \right) \frac{\delta_R}{16} , \\ d_8^R &= \frac{1}{4} \left(\frac{17}{9} D_R^2 + F_R^2 \right) \delta_R , \end{aligned} \quad (55)$$

in terms of the parameter β_R to be fixed below.

4.3 Scalar meson contribution to the low–energy constants

Meson–exchange in the t –channel can also contribute to the LECs as pointed out in [16]. Denoting by S and S_1 the scalar octet and singlet with $M_S \simeq M_{S_1} \simeq 1$ GeV, respectively, the lowest order effective Lagrangian coupling the scalars to the Goldstone bosons and to

the baryon octet reads

$$\begin{aligned}
\mathcal{L}_{\text{eff}}(U, B, S) &= \mathcal{L}_0(S) + \mathcal{L}_{\text{int}}(U, S) + \mathcal{L}_{\text{int}}(B, S) , \\
\mathcal{L}_0(S) &= \frac{1}{2} \text{Tr} (\partial_\mu S \partial^\mu S - M_S^2 S^2) + \frac{1}{2} \text{Tr} (\partial_\mu S_1 \partial^\mu S_1 - M_{S_1}^2 S_1^2) , \\
\mathcal{L}_{\text{int}}(U, S) &= c_d \text{Tr}(S u_\mu u^\mu) + c_m \text{Tr}(S \chi_+) + \tilde{c}_d S_1 \text{Tr}(u_\mu u^\mu) + \tilde{c}_m S_1 \text{Tr}(\chi_+) , \\
\mathcal{L}_{\text{int}}(B, S) &= D_S \text{Tr}(\bar{B}\{S, B\}) + F_S \text{Tr}(\bar{B}[S, B]) + D_{S_1} S_1 \text{Tr}(\bar{B}B)
\end{aligned} \tag{56}$$

where the coupling constants D_S , F_S and D_{S_1} have chiral dimension zero. For the couplings of the scalars to the Goldstone bosons, we use the notation of [31] and the parameters determined therein, i.e. $|c_d| = 42 \text{ MeV}$, $|c_m| = 32 \text{ MeV}$ with $c_d c_m > 0$. The singlet couplings can be related to these in the large- N_c limit via $\tilde{c}_{d,m} = \pm c_{d,m}/\sqrt{3}$ with $\tilde{c}_d \tilde{c}_m > 0$. Similarly, we have $D_{S_1} = \pm 2D_S/\sqrt{3}$. In fact, it is very difficult to pin down the couplings D_S , F_S and D_{S_1} .^{#8} We will therefore leave F_S and D_S as free parameters and determine them as described below.

Evaluating now the diagrams shown in Fig.4, which are scattering graphs and direct couplings of the scalar mesons to the operator χ_+ , and using the lowest order effective Lagrangian Eq.(56), one finds only contributions to $\mathcal{L}_{\phi B}^{(2)}$. Contributions to the LECs d_i arise if one accounts for higher dimension operators in $\mathcal{L}_{\text{int}}(B, S)$. This would amount to a proliferation of unknown coupling constants. We have refrained from including such terms based on the observation that the scalar contribution to the dimension two LECs is rather small [16]. This amounts to

$$\begin{aligned}
b_0^S &= -\frac{2}{3} C_m D_S + \tilde{C}_m D_{S_1} , \quad b_D^S = C_m D_S , \quad b_F^S = C_m F_S , \\
b_1^S &= \frac{1}{2} C_d D_S , \quad b_2^S = C_d F_S , \quad b_3^S = \frac{1}{2} C_d D_S , \quad b_8^S = -\frac{2}{3} C_d D_S + \tilde{C}_d D_{S_1} ,
\end{aligned} \tag{57}$$

with

$$C_{m,d} = \frac{c_{m,d}}{M_S^2} , \quad \tilde{C}_{m,d} = \frac{\tilde{c}_{m,d}}{M_{S_1}^2} . \tag{58}$$

We have now assembled all pieces to discuss the numerical values of the various LECs.

4.4 Determination of the low-energy constants

For pinning down the numerical values of the various LECs, we need to know the values of the parameters β_Δ , δ_Δ , β_R , F_S , D_S and of the baryon octet mass in the chiral limit, \hat{m} . We will later show that it is mandatory to keep all these parameters, i.e. using only the decuplet and/or the scalar mesons to estimate the LECs does not lead to a consistent description of the baryon masses and σ -terms. We determine these parameters as follows.

^{#8}In what follows, we neglect the singlet fields. Their effects can be absorbed in the pertinent coupling constant of the octet fields.

Within the framework of resonance exchange saturation of the LECs , the baryon masses take the form

$$m_B = \overset{\circ}{m} + m_B^{(3)} + \frac{1}{m_B} \lambda_B + \beta D_B^\beta + \delta D_B^\delta + \epsilon D_B^\epsilon + D_B^S , \quad (59)$$

where we have lumped the $\mathcal{O}(m_q)$ and the $\mathcal{O}(m_q^2)$ corrections together (in the constants D_B^β and D_B^S , respectively). We remark that whenever possible, we have substituted the octet mass in the chiral limit by the corresponding physical mass since these differences are of higher order. The λ contributions are the $1/m$ insertions from $\mathcal{L}_{\phi B}^{(2)}$. Similarly, the β and ϵ terms stem from tadpole graphs with insertions proportional to the low-energy constants $b_{0,D,F}$ and b_i , respectively. The δ terms subsume the contributions from $\mathcal{L}_{\phi B}^{(4)}$, these are proportional to the low-energy constants d_i . In these last three terms, we have abbreviated

$$\begin{aligned} \beta D_B^\beta &= \beta_\Delta D_{B,\Delta}^\beta + \beta_R D_{B,R}^\beta , & \delta D_B^\delta &= \delta_\Delta D_{B,\Delta}^\delta + \delta_R D_{B,R}^\delta , \\ \epsilon D_B^\epsilon &= \epsilon_\Delta D_{B,\Delta}^\epsilon + \epsilon_R D_{B,R}^\epsilon . \end{aligned} \quad (60)$$

Finally, the terms of the type D_B^S are the scalar meson contributions to the mass (which could also be absorbed in the coefficients β and ϵ , cf. Eq.(57)). The numerical values of the λ_B , D_B^β , D_B^δ , and D_B^ϵ are given in table 1, using $F = 0.5$, $D = 0.75$, $D_R = 0.60$, $F_R = 0.11$ and $F_\pi = 100$ MeV. We see that the dominant terms at $\mathcal{O}(m_q^2)$ are indeed the tadpole graphs with an insertion from $\mathcal{L}_{\phi B}^{(2,br)}$ (this holds for the masses but not for $\sigma_{\pi N}(0)$). These numbers are different from the ones given in Ref.[16] due to the different definitions of the coefficients β , δ and ϵ and because of the Roper-octet contribution. The D_B^S are (in GeV)

$$\begin{aligned} D_N^S &= -0.013 D_S + 0.057 F_S , & D_\Lambda^S &= -0.042 D_S - 0.019 F_S , \\ D_\Sigma^S &= +0.052 D_S + 0.019 F_S , & D_\Xi^S &= -0.045 D_S - 0.076 F_S , \end{aligned} \quad (61)$$

where we have used the results of [31] for the scalar couplings and $M_S = 0.983$ GeV. The four equations for the octet masses

$$m_B = m_B[\overset{\circ}{m}, \beta_\Delta, \delta_\Delta, \beta_R, F_S, D_S] , \quad (62)$$

allow to pin down four parameters, we choose δ_Δ , β_Δ , F_S and D_S . A similar equation can be worked out for the pion-nucleon σ -term,

$$\begin{aligned} \sigma_{\pi N}(0) &= \sigma_{\pi N}^{(3)}(0) + \frac{1}{m_N} \lambda_\sigma + \beta_\Delta D_\sigma^{\beta\Delta} + \beta_R D_\sigma^{\beta R} + \delta_\Delta D_\sigma^{\delta\Delta} + \delta_R D_\sigma^{\delta R} \\ &\quad + \epsilon_\Delta D_\sigma^{\epsilon\Delta} + \epsilon_R D_\sigma^{\epsilon R} + D_S D_\sigma^{D_S} + F_S D_\sigma^{F_S} , \end{aligned} \quad (63)$$

or numerically with $\sigma_{\pi N}(0) = 45$ MeV

$$0.077 = -0.244 \beta_\Delta + 0.020 \delta_\Delta - 0.056 \beta_R + 0.001 D_S - 0.002 F_S , \quad (64)$$

in appropriate units of GeV. We end up with 5 equations for six unknown parameters. Choosing β_R as the undetermined one, this leads to

$$\begin{aligned}\overset{\circ}{m} &= +0.7673 + 0.3737 \beta_R, \quad \beta_\Delta = -0.3321 - 0.1126 \beta_R, \\ \delta_\Delta &= -0.1387 - 0.0232 \beta_R, \quad D_S = -5.8244 + 1.4484 \beta_R, \\ F_S &= -0.8974 - 12.715 \beta_R.\end{aligned}\tag{65}$$

Units are GeV for $\overset{\circ}{m}$, GeV^{-1} for β_Δ, β_R and GeV^{-3} for δ_Δ whereas F_S and D_S are dimensionless. The prefactors have the appropriate dimensions, for example $\overset{\circ}{m} [\text{GeV}] = 0.767 [\text{GeV}] + 0.374 [\text{GeV}^2] \beta_R [\text{GeV}^{-1}]$. We can now give a bound on β_R based on the assumption that the ratio of the decuplet contribution and of the Roper octet one is of the same order for all LECs. Since the contributions to the b_i do not contain any unknown parameters, we can use their ratio to estimate the corresponding one from the Roper–octet and we thus conclude

$$\left| \frac{D_{B,R}^\beta \beta_R}{D_{B,\Delta}^\beta \beta_\Delta} \right| \sim \left| \frac{D_{B,R}^\epsilon \epsilon_R}{D_{B,\Delta}^\epsilon \epsilon_\Delta} \right| \tag{66}$$

from which we derive the (soft) bound

$$|\beta_R| \leq 0.1 \text{ GeV}^{-1} . \tag{67}$$

Consequently, the scalar couplings F_S and D_S are falling into the ranges $-2 < F_S < 0.5$ and $-6.0 < D_S < -5.7$. It would be interesting to have some phenomenological bounds on these couplings as a check of the consistency of our procedure.

Finally, we can now express the LECs in terms of the parameter β_R ,

$$\begin{aligned}b_0 &= -0.606 - 0.227 \beta_R, \quad b_D = +0.079 + 0.014 \beta_R, \quad b_F = -0.316 - 0.536 \beta_R, \\ b_1 &= -0.004 + 0.024 \beta_R, \quad b_2 = -0.187 - 0.421 \beta_R, \\ b_3 &= +0.018 + 0.024 \beta_R, \quad b_8 = -0.109 - 0.032 \beta_R, \\ d_1 &= +0.008 + 0.001 \beta_R, \quad d_2 = +0.035 + 0.006 \beta_R, \\ d_3 &= +0.069 + 0.012 \beta_R, \quad d_4 = -0.077 - 0.013 \beta_R, \\ d_5 &= -0.050 - 0.008 \beta_R, \quad d_7 = -0.018 - 0.003 \beta_R, \quad d_8 = -0.103 - 0.017 \beta_R, \end{aligned} \tag{68}$$

with the b_i in GeV^{-1} and the d_i in GeV^{-3} and the coefficients have corresponding dimensions. In table 2, we compare the LECs of the dimension two meson–baryon effective Lagrangian with the values previously determined from KN scattering data. We have transformed the results of Ref.[42] into our notation. As can be seen from table 2, most (but not all) coefficients agree in sign and magnitude. We remark that the procedure used in [42] involves the summation of arbitrary high orders via a Lippmann–Schwinger equation and is thus afflicted with some uncertainty not controlled within CHPT.^{#9} For comparison, we also give the values of $b_{0,D,F}$ from the q^3 calculation [7].

^{#9}These values have recently been refitted taking account also η and kaon photoproduction data [43]. The resulting values are somewhat different from the ones given in ref.[42].

We end this paragraph with some comments on estimating the LECs from the decuplet or decuplet and Roper–octet alone. Consider first the case of the decuplet only. We then have three parameters determined by a least-square fit to the four masses. One finds $\mathring{m} = 620 \text{ MeV}$, $\beta_\Delta = -0.551 \text{ GeV}^{-1}$ and $\delta_\Delta = 0.975 \text{ GeV}^{-3}$ and the deviations of the fitted from the physical masses range from -154 to 121 MeV. This means that in particular the deviation from the Gell-Mann–Okubo relation,

$$\Delta_{\text{GMO}} = \frac{1}{4}(3m_\Lambda + m_\Sigma - 2m_N - 2m_\Xi) \quad , \quad (69)$$

which empirically is about 6.5 MeV, is very large (of the order of 200 MeV). The LECs take values which are considerably larger than the ones given above (cf. table 2 and Eq.(68)). Furthermore, the σ -term is completely fixed and turns out to be $\sigma_{\pi N}(0) = 89 \text{ MeV}$, considerably larger than the empirical value. One could try to remedy the situation by adding the Roper–octet, thus having one more free parameter at ones disposal. One can then solve the linear system of equations for the four masses in terms of the four parameters. This does, however, not lead to sensible results. One finds $\beta_R \simeq 10 \text{ GeV}^{-1}$, much larger than the bound given in Eq.(67). This leads to absurd results like $\mathring{m} \simeq 3.9 \text{ GeV}$. It is therefore mandatory to include the scalar meson exchange to consistently describe the scalar sector of three flavor baryon CHPT.

4.5 Two-loop contributions to the LECs

As shown, the estimation of the LECs entering the baryon masses involves Goldstone boson loop diagrams, compare Fig. 3. With such graphs, one encounters a new mass scale which is non-vanishing in the chiral limit, $\mathring{m}_R - \mathring{m} \neq 0$ as $M_\phi \rightarrow 0$. Therefore, a strict one-to-one correspondence between the expansion in small momenta and the number of Goldstone boson loops is no longer guaranteed, as it is known from the study of relativistic baryon CHPT [36]. A similar situation arises for the calculation of the deviations from Dashen’s theorem, which involves photons coupled to heavy (axial-)vector mesons in the loops, see e.g. Refs.[44]. Again, the new mass scale (here the vector meson mass in the chiral limit) spoils the strict power counting. Indeed, it has been shown that two-loop graphs modify the leading order results to the deviations from Dashen’s theorem [45]. We therefore also have to address this issue in our context.

The possible two-loop graphs to be considered are shown in Fig. 5. Here, we concentrate on the diagrams with the maximal number of resonance propagators and the simplest $BB^*\phi$ coupling via the chiral connection Γ_μ . This means for example in diagram (e) one of the intermediate propagators is a nucleon and the other one refers to an excited intermediate spin-3/2 or spin-1/2 state. For all these graphs, all possible combinations of decuplet, Roper–octet and nucleon propagators should be considered. From these, we only take the leading ones in the large mass limit subject to the constraint that the vertices involved are constructed from the chiral connection. The corresponding contributions from the axial-vector couplings $\sim u_\mu$ involve other coupling constants and are therefore algebraically independent from the ones considered here. In fact, to simplify the algebra,

we will focus on the case where all resonance propagators refer to the Roper-octet. A more detailed account of these calculations is given in [30].

Consider first the tadpole-type diagrams like in Fig.5a. Without Clebsch-Gordan coefficients and other prefactors, it takes the form

$$\begin{aligned}
I_{5a} &= \int \frac{d^4 l}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \frac{i}{l^2 - M_a^2} (\not{l} - \not{k}) \gamma_5 \frac{i}{\not{k} - m_R} (\not{l} - \not{k}) \gamma_5 \frac{i}{(p - k)^2 - M_b^2} \\
&= -M_a^2 \left[2L + \frac{1}{16\pi^2} \ln \frac{M_a^2}{\lambda^2} \right] \left\{ im_R (m_R^2 + M_b^2) \left[2L + \frac{1}{16\pi^2} \ln \frac{m_R^2}{\lambda^2} \right] \right. \\
&\quad \left. + \frac{i}{32\pi^2} \frac{\overset{\circ}{m}}{m_R^2} M_b^4 - \frac{i}{16\pi^2 m_R} M_b^4 \ln \frac{M_b^2}{\lambda^2} \right\} + \dots, \tag{70}
\end{aligned}$$

where we have only retained the leading terms in the expansion in the resonance mass m_R as indicated by the ellipsis, i.e. we have approximated $m_R - \overset{\circ}{m} \simeq m_R$ and so on. We remark that the disturbing term $\sim M_a^2 \ln M_a^2$ cancels in the sum of graphs 5a, its partner with the tadpole at the other vertex and graph 5f. Taking into account the pertinent counter term graphs to renormalize the infinities connected to the tadpole, we find

$$I_{5a} \sim M_a^2 \left\{ im_R (m_R^2 + M_b^2) \left[2L + \frac{1}{16\pi^2} \ln \frac{m_R^2}{\lambda^2} \right] + \mathcal{O}(M_b^4) \right\}. \tag{71}$$

The first term in Eq.(71) can be completely absorbed in the renormalization constant $\alpha_R^{(2)}$ and the second one leads to a new renormalization constant $\alpha_R^{(4)}$. Its finite piece modifies the value of the finite dimension four one-loop contribution δ_R . Similarly, for graphs with intermediate decuplet states, to leading order in the resonance mass one can absorb the two-loop contribution entirely in a redefinition of the constants $\alpha_\Delta^{(2)}$ and $\alpha_\Delta^{(4)}$. Furthermore, if one works out the first finite contribution at orders $M_a^{2,4}$, one finds that these are numerically much smaller than the corresponding pieces from the one loop diagram. Similarly, we work out the contribution from the graphs of the type 5d. The integral stripped off all prefactors and expanded in powers of the resonance mass takes the form

$$\begin{aligned}
I_{5d} &= -2m_R^3 \left[2L + \frac{1}{16\pi^2} \ln \frac{m_R^2}{\lambda^2} \right]^2 (M_a^2 + M_b^2) \\
&\quad - 2m_R \frac{1}{16\pi^2} \ln \frac{m_R^2}{\lambda^2} \left[2L + \frac{1}{16\pi^2} \ln \frac{m_R^2}{\lambda^2} \right] (M_a^4 + M_b^4) + \dots \tag{72}
\end{aligned}$$

Again, standard renormalization has to be performed and one is left with a contribution to $\alpha_\Delta^{(2)}$ and one to $\alpha_\Delta^{(4)}$. The latter can be made to vanish if one sets $\lambda = m_R$.

The other diagrams shown in Fig.5 can not be given in closed analytical form. To get an estimate about their contributions, we perform asymptotic expansions in the external momentum p making use of the formalism developed in Ref.[46] (and references therein). For external momenta (here $p = \overset{\circ}{m}$) below the first threshold (here $p_{\text{thr}} = \overset{\circ}{m} + 2M_\pi$), one can expand around $p = 0$ to leading order. It is straightforward to show [30] that this

procedure is sufficient to estimate the leading order terms in the large resonance mass expansion (here m_R). Specifically, we have

$$I_{5\alpha}(p^2) = I_{5\alpha}(0) + \frac{p^2}{2d} \square_p I_{5\alpha}(p^2) + \mathcal{O}(p^4) \quad (\alpha = b, c, e) , \quad (73)$$

in d space–time dimensions. The explicit formulae for the various contributions are very lengthy and can be found in [30]. Here, we just show a typical result after picking out the leading terms in the m_R expansion,

$$I_{5c} = e_1 m_R^5 + e_2 m_R^3 (M_a^2 + M_b^2) + e_3 m_R (M_a^4 + M_b^4) + e_4 m_R M_a^2 M_b^2 + \dots , \quad (74)$$

with $\epsilon = d - 4$ and the coefficients e_i contain divergences. After removing the divergent pieces $\sim 1/\epsilon$ and $1/\epsilon^2$, we are essentially left with contributions to the renormalization constants $\alpha_\Delta^{(2,4)}$ and $\alpha_R^{(2,4)}$, i.e. the two–loop effects can be completely absorbed in pertinent redefinitions. Only the finite one–loop constant δ_R is modified. The corresponding tree graph contributions to the baryon masses and σ –terms at order M_ϕ^4 from $\mathcal{L}_{\phi B}^{(4)}$ are, however, very small and thus an accurate knowledge of this coupling is not needed. From this we conjecture that a similar mechanism is also operative at higher orders and that our one–loop approach to estimate the baryon resonance excitations to the various LECs is a consistent procedure.

4.6 Some two–loop contributions to the baryon masses

Having worked out the two–loop contributions to the LECs b_i and d_i in paragraph 4.5, we can use the same machinery to estimate some typical two–loop, i.e. order q^5 , contributions to the baryon masses. This should only be considered indicative since we do not attempt a full $\mathcal{O}(q^5)$ calculation here, which besides all two–loop diagrams would also involve one–loop graphs with exactly one insertion from $\mathcal{L}_{\phi B}^{(3)}$.

Consider first the tadpole–type graph like in Fig.5a (and its partner with the tadpole on the other side) but with the essential difference that the intermediate propagator refers to a groundstate spin–1/2 state in the heavy baryon formalism. Using the appropriate Feynman rules, the momentum–space integral I_t is

$$I_t = \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \frac{i}{q^2 - M_a^2} \frac{i}{k^2 - M_b^2} \frac{i}{v \cdot (p - k)} (S \cdot k)^2 . \quad (75)$$

To leading order, we can neglect the baryon off–shell momentum. I_t then takes the simple form

$$I_t = \frac{1}{4} f(M_a) M_b^2 \int \frac{d^4 k}{(2\pi)^4} \frac{-1}{[k^2 - M_b^2] v \cdot k} = \frac{-i}{32\pi} M_a^2 M_b^3 \left[2L + \frac{1}{16\pi^2} \ln \frac{M_a^2}{\lambda^2} \right] . \quad (76)$$

As expected from the power counting, this contribution starts at order M_ϕ^5 . Renormalizing the divergence and restoring the appropriate prefactors, we have the following

contribution to the nucleon mass from these tadpole-type graphs

$$m_N^{(5,a)} = \frac{45F^2 + 5D^2 - 6DF}{3072 \pi^3 F_\pi^4} M_K^5 \ln \frac{M_K^2}{\lambda^2} , \quad (77)$$

where we have set for simplification $M_\pi = 0$ and $M_K = M_\eta$. Similarly, we can work out the contribution from the analog of fig. 5f with heavy baryon propagators,

$$m_N^{(5,f)} \simeq -\frac{45F^2 + 17D^2 - 30DF}{768 \pi^3 F_\pi^4} M_K^5 \ln \frac{M_K^2}{\lambda^2} . \quad (78)$$

The “double-hump” graph (compare fig. 5d) vanishes to this order, $m_N^{(5,d)} = 0$. Eqs.(77,78) will be used in the next section to get an order of magnitude estimate of the two-loop corrections to the nucleon mass.

5 Results and discussion

5.1 Results for the central values of resonance parameters

In this section, we discuss the results for our central values of parameters, setting $\beta_R = 0$ (varying β_R within its allowed range, $-0.1 \leq \beta_R \leq 0.1 \text{ GeV}^{-1}$, does only lead to irrelevant changes). These are $F_\pi = 100 \text{ MeV}$, $D = 0.75$, $F = 0.5$, $\mathcal{C} = 1.5$, $D_R = 0.6$, $F_R = 0.11$ and we set $\lambda = 1 \text{ GeV}$. The various mesonic LECs $L_i^r(\lambda)$ are the central values taken from the compilation of Bijnens et al. in Ref.[47]. For these central values, all baryon masses are fitted exactly, together with $\sigma_{\pi N}(0) = 45 \text{ MeV}$. The theoretical uncertainties induced by the spread of these parameters, in particular due to the $\pm 10 \text{ MeV}$ uncertainty in $\sigma_{\pi N}(0)$ [22], will be discussed in the next subsection. We find for the octet baryon mass in the chiral limit using Eq.(65)

$$\overset{\circ}{m} = 767 \text{ MeV} . \quad (79)$$

The quark mass expansion of the baryon masses, in the notation of Eq.(16), reads

$$\begin{aligned} m_N &= \overset{\circ}{m} (1 + 0.34 - 0.35 + 0.24) , \\ m_\Lambda &= \overset{\circ}{m} (1 + 0.69 - 0.77 + 0.54) , \\ m_\Sigma &= \overset{\circ}{m} (1 + 0.81 - 0.70 + 0.44) , \\ m_\Xi &= \overset{\circ}{m} (1 + 1.10 - 1.16 + 0.78) . \end{aligned} \quad (80)$$

We observe that there are large cancellations between the second order and the leading non-analytic terms of order q^3 , a well-known effect. The fourth order contribution to the nucleon mass is fairly small, whereas it is sizeable for the Λ , the Σ and the Ξ . This is partly due to the small value of $\overset{\circ}{m}$, e.g. for the Ξ the leading term in the quark mass expansion gives only about 55% of the physical mass and the second and third order terms cancel

almost completely. The fourth order contributions are indeed dominated by the one-loop graphs with insertions from $\mathcal{L}_{\phi B}^{(2,\text{br})}$ as conjectured by Jenkins and Manohar [5, 6]. However, one can not neglect the terms with insertions from the remaining dimension two terms, which are proportional to the b_i and stem from relativistic $1/m$ corrections. In contrast, the contributions from the local terms $\mathcal{L}_{\phi B}^{(4)}$ are fairly small, i.e. one does not need to know the LECs d_i very accurately. From the chiral expansions exhibited in Eq.(80) one can not yet draw a final conclusion about the rate of convergence in the three-flavor sector of baryon CHPT. Certainly, the breakdown of CHPT claimed in ref.[3] is not observed. On the other hand, the conjecture [21] that only the leading non-analytic corrections (LNAC) $\sim m_q^{3/2}$ are large and that further terms like the ones $\sim m_q^2$ are moderately small, of the order of 100 MeV, is not supported by our findings.

We now turn to the σ -terms and the strangeness content of the nucleon. The pion-nucleon σ -term is used in the fitting procedure. It is, however, instructive to disentangle the various contributions to $\sigma_{\pi N}(0)$ of order q^2 , q^3 and q^4 , respectively,

$$\sigma_{\pi N}(0) = 58.3 (1 - 0.56 + 0.33) \text{ MeV} = 45 \text{ MeV} , \quad (81)$$

which shows a moderate convergence, i.e. the terms of increasing order become successively smaller. Still, the q^4 contribution is important. Also, at this order no $\pi\pi$ rescattering effects are included. Rewriting the σ -term as [3]

$$\sigma_{\pi N}(0) = \frac{\hat{\sigma}}{1 - y} \quad (82)$$

we find for the strangeness fraction y and for $\hat{\sigma}$

$$y = 0.21 , \quad \hat{\sigma} = 36 \text{ MeV} . \quad (83)$$

The value for y is within the band deduced in ref.[22], $y = 0.15 \pm 0.10$ and the value for $\hat{\sigma}$ compares favourably with Gasser's estimate, $\hat{\sigma} = 33 \pm 5 \text{ MeV}$ [3].

Finally, we consider the kaon-nucleon σ -terms and the various scalar form factors. As stressed in section 3.3, there appear undetermined renormalization constants at order q^4 as long as one works in the isospin limit $m_u = m_d$. These are expected to be of order one. Indeed, for the pion-nucleon σ -term one can calculate this constant (called a_π) since the full renormalization has been performed ($\beta_R = 0$),

$$\sigma_{\pi N}(0) = (50.1 - 14.4 a_\pi) \text{ MeV} \rightarrow a_\pi = 0.36 , \quad (84)$$

i.e. a_π has indeed the expected size. For the kaon-nucleon σ -terms, we can give the results up to two constants (called a_{K1} and a_{K2}),

$$\begin{aligned} \sigma_{KN}^{(1)}(0) &= (369 - 306 a_{K1}) \text{ MeV} , \\ \sigma_{KN}^{(2)}(0) &= (934 - 437 a_{K2}) \text{ MeV} , \end{aligned} \quad (85)$$

with the respective chiral expansions

$$\begin{aligned} \sigma_{KN}^{(1)}(0) &= (528 - 524 + 365 - 306 a_{K1}) \text{ MeV} , \\ \sigma_{KN}^{(2)}(0) &= (290 - 49 + 693 - 437 a_{K2}) \text{ MeV} , \end{aligned} \quad (86)$$

where the terms refer to the orders q^2 , q^3 , q^4 and q^4 , respectively (the q^4 contribution independent of the renormalization constant a_{Ki} is shown separately). These numbers agree with the q^3 calculation of ref.[7] (to that order). Varying a_{K1} and a_{K2} between 0.5 and 1, we have

$$\sigma_{KN}^{(1)}(0) = 73 \dots 216 \text{ MeV} , \quad \sigma_{KN}^{(2)}(0) = 493 \dots 703 \text{ MeV} . \quad (87)$$

These numbers should be considered indicative and can only be sharpened in a calculation with $m_u \neq m_d$. For a discussion of the various σ -term shifts to the pertinent Cheng–Dashen points, we refer to ref.[30].

5.2 Theoretical uncertainties

In the previous paragraph, we gave the results for the central values of the input parameters. Here, we will discuss the spread of the results due to the uncertainties related to these numbers.

Consider first the dependence on the coupling constant \mathcal{C} and the values of the axial–vector couplings F and D . For comparison with our central values, we also use $|\mathcal{C}| = 1.2$ determined by Butler et al. [48], $|\mathcal{C}| = 1.8$ from the decay $\Delta \rightarrow N\gamma$ and $D = 0.85 \pm 0.06$, $F = 0.52 \pm 0.04$ given by Luty and White [49]. The results depend only very weakly on these parameters, i.e. they vary within a few percent. For the case of \mathcal{C} , this weak dependence stems from the fact that \mathcal{C} only changes the value of ϵ_Δ whereas the changes in the much more important β_Δ and δ_Δ are absorbed in the new fit values of $\alpha_\Delta^{(2,4)}$. The weak dependence on the actual values of F and D is due to compensating contributions of third and fourth order and the already mentioned dominance of the tadpole graphs with the symmetry breakers $\sim b_{0,D,F}$. Consider now variations in the renormalization scale λ . The latter dependence is induced since we estimate the LECs from resonance exchange and would disappear once all LECs could be determined from data. In table 3 we show results for the range $0.8 \text{ GeV} \leq \lambda \leq 1.2 \text{ GeV}$, for the central values of F_π , F , D , F_R and D_R . The strangeness fraction y is most notably affected. The chiral series for the masses converges quicker for lower values of λ . In table 4, we vary the value of $\sigma_{\pi N}(0)$ generously by $\pm 10 \text{ MeV}$. Again, the strangeness fraction shows the largest variation. All these variations are essential linear in $\delta\sigma_{\pi N}(0)$. Finally, we remark that varying the pseudoscalar decay constant F_π between 93 and 113 MeV does also not alter any of the previous numbers drastically. We therefore assign the following theoretical uncertainty to the results for the average octet mass in the chiral limit, the strangeness fraction y and $\hat{\sigma}$, in order

$$\overset{\circ}{m} = 767 \pm 110 \text{ MeV} , \quad y = 0.21 \pm 0.20 , \quad \hat{\sigma} = 36 \pm 7 \text{ MeV} . \quad (88)$$

These uncertainties do not include the possible effects of higher orders which can only be assessed if one performs such a calculation. As an indication of genuine two loop contributions, we quote the results of the diagrams evaluated in paragraph 4.6. We find (setting $\lambda = 1 \text{ GeV}$)

$$m_N^{(5a)} = -52 \text{ MeV} \quad m_N^{(5f)} = -13 \text{ MeV} , \quad (89)$$

which are individually very small, i.e. well within the uncertainties discussed above. It is expected that the major contribution at order q^5 does indeed come from one-loop graphs with insertions from $\mathcal{L}_{\pi N}^{(3)}$ and not from the genuine two loop diagrams. To quantify this statement, such an order q^5 calculation has to be performed. That, however, goes beyond the scope of the present paper.

6 Summary and outlook

In this paper, we have considered the chiral expansion of the groundstate baryon octet baryon masses and the pion-nucleon σ -term to quadratic (fourth) order in the quark (Goldstone boson) masses, in the framework of heavy baryon chiral perturbation theory. The pertinent results of this investigation can be summarized as follows:

- We have constructed the most general effective Lagrangian to fourth order in the small parameter q (external momentum or meson mass) necessary to investigate the scalar sector. Besides the standard dimension two symmetry-breaking terms, Eq.(8), it contains further dimension two operators with derivatives acting on the Goldstone boson fields and kinematical $1/m$ corrections, cf. Eq.(9) and Eq.(13).
- We have given the complete expressions for the baryon masses ($m_{N,\Lambda,\Sigma,\Xi}$) at order q^4 together with the pion-nucleon σ -term. At this order, divergences appear (in contrast to $\mathcal{O}(q^3)$, where the one loop corrections to the masses and $\sigma_{\pi N}(0)$ are finite). The renormalization procedure to render the baryon self-energies and also $\sigma_{\pi N}(0)$ finite (by use of the Feynman-Hellmann theorem) involves additional terms as listed in Eq.(28). The renormalization of the kaon-nucleon σ -terms and the corresponding scalar form factors is further complicated by additional momentum-(in)dependent divergences, as detailed in appendix B.
- There appear seven low-energy constants (LECs) at order q^2 (called b_i) and seven more at order q^4 (called d_i) to calculate the masses and pion-nucleon σ -term. Two of the latter amount to quark mass renormalizations of two of the b_i . Since there do not exist enough data to fix all these, we have estimated them by means of resonance exchange. Besides standard tree graphs with scalar meson exchange, this involves Goldstone boson loops with intermediate baryon resonances (spin-3/2 decuplet and the spin-1/2 (Roper) octet) for the scalar-isoscalar LECs. We have discussed a consistent scheme how to implement resonance exchange under such circumstances, i.e. which avoids double-counting and abides to the strictures from analyticity. Within the one-loop approximation and to leading order in the resonance masses, the analytic pieces of the pertinent graphs are still divergent, i.e. one is left with three a priori undetermined renormalization constants (β_Δ , δ_Δ and β_R). These have to be determined together with the finite scalar couplings F_S and D_S and the octet mass in the chiral limit. Using the baryon masses and the value of $\sigma_{\pi N}(0)$ as input, we can determine all LECs in terms of one parameter, β_R . We derive a bound on this parameter and show that observables are insensitive to

variations of β_R within its allowed range. We have also demonstrated that the effects of two (and higher) loop diagrams can almost entirely be absorbed in a redefinition of the one loop renormalization parameters.

- Within this scheme of estimating the LECs we determine the baryon mass in the chiral limit, denoted $\overset{\circ}{m}$, $\overset{\circ}{m} = 770 \pm 110$ MeV (accounting also for the uncertainty in certain input parameters like e.g. the pion–nucleon σ –term). For the strangeness fraction y we find $y = 0.21 \pm 0.20$, consistent with dispersion–theoretical determinations. This translates into $\hat{\sigma} = 36 \pm 7$ MeV, compare Eq.(82), in good agreement with a previous calculation [3].
- The chiral expansions for the nucleon mass and the pion–nucleon σ –term are moderately well behaved whereas for the hyperon masses $m_{\Lambda, \Sigma, \Xi}$ there still appear sizeable corrections at order q^4 . This is partly due to the almost complete cancelations between the terms of order m_q and $m_q^{3/2}$ and the smallness of the baryon mass in the chiral limit. A definite statement about the convergence of three–flavor baryon CHPT can thus not yet be made.
- We have also estimated the two kaon–nucleon σ –terms, which take the ranges given in Eq.(86) based on dimensional analysis for the appearing renormalization constants $a_{K1, K2}$.

Acknowledgements

We are grateful to Jürg Gasser and Joachim Kambor for some very useful comments. A series of discussions with various participants of the workshop “The Standard Model at Low Energies” at the ECT* in Trento in May 1996 helped to sharpen our views. We are grateful to the ECT* for offering this opportunity.

A Baryon mass coefficients at order q^4

Here, we give the coefficients $\epsilon_{i,B}^{P(Q)}$ with $i = 1, 2, 3, 4$ and $P, Q = \pi, K, \eta$ for the Λ , the Σ and the Ξ . All d_i are understood as $d_i^r(\lambda)$.

Λ :

$$\begin{aligned}
\epsilon_{1,\Lambda}^{\pi} &= -4(4d_3 + \frac{8}{3}d_4 + d_7 + 3d_8) , \epsilon_{1,\Lambda}^K = -16(\frac{8}{3}d_3 + \frac{2}{3}d_4 + d_7 + d_8) , \\
\epsilon_{2,\Lambda}^{\pi K} &= 16(-d_7 + d_8 + \frac{8}{3}d_3 + \frac{4}{3}d_4) , \epsilon_{3,\Lambda}^{\pi} = 2b_D + 6b_0 - 4b_3 - 6b_8 - \frac{D^2}{\overset{\circ}{m}} , \\
\epsilon_{3,\Lambda}^K &= \frac{20}{3}b_D + 8b_0 - 12b_1 - \frac{4}{3}b_3 - 8b_8 - \frac{D^2 + 9F^2}{3\overset{\circ}{m}} , \\
\epsilon_{3,\Lambda}^{\eta} &= 2b_0 - 4b_3 - 2b_8 - \frac{D^2}{3\overset{\circ}{m}} , \epsilon_{4,\Lambda}^{\pi\pi} = -16D^2b_D , \epsilon_{4,\Lambda}^{\pi K} = 16D^2b_D ,
\end{aligned}$$

$$\begin{aligned}
\epsilon_{4,\Lambda}^{K\pi} &= -(D^2 + 9F^2) \left(\frac{4}{3}b_d - 2b_0 \right) - (D - 3F)^2 (-2b_F + b_0) \\
&\quad - (D + 3F)^2 (2b_F + b_0) , \\
\epsilon_{4,\Lambda}^{KK} &= (D^2 + 9F^2) \left(\frac{16}{3}b_D + 4b_0 \right) - 2(D - 3F)^2 (b_0 + b_D + b_F) \\
&\quad - 2(D + 3F)^2 (b_0 + b_D - b_F) , \\
\epsilon_{4,\Lambda}^{\eta\pi} &= -\frac{14}{9}b_D , \quad \epsilon_{4,\Lambda}^{\eta K} = \frac{32}{9}b_D .
\end{aligned} \tag{A.1}$$

$\Sigma :$

$$\begin{aligned}
\epsilon_{1,\Sigma}^{\pi} &= -4(4d_3 + d_7 + 3d_8) , \quad \epsilon_{2,\Sigma}^{\pi K} = 16(-d_7 + d_8) , \quad \epsilon_{1,\Sigma}^K = -16(d_7 + d_8) , \\
\epsilon_{3,\Sigma}^{\pi} &= 6b_D + 6b_0 - 8b_1 - 4b_3 - 6b_8 - \frac{D^2 + 6F^2}{3\overset{\circ}{m}} , \\
\epsilon_{3,\Sigma}^K &= 4b_D + 8b_0 - 4b_1 - 4b_3 - 8b_8 - \frac{5D^2 + 6DF + 9F^2}{6\overset{\circ}{m}} , \\
\epsilon_{3,\Sigma}^{\eta} &= 2b_0 - \frac{4}{3}b_3 - 2b_8 - \frac{D^2}{3\overset{\circ}{m}} , \quad \epsilon_{4,\Sigma}^{\pi K} = \frac{16}{3}D^2 b_D , \\
\epsilon_{4,\Sigma}^{\pi\pi} &= \left(\frac{2}{3}D^2 + 4F^2 \right) (6b_D + 3b_0) - 4D^2 \left(-\frac{1}{3}b_D + \frac{1}{2}b_0 \right) - 12(2b_D + b_0) , \\
\epsilon_{4,\Sigma}^{K\pi} &= 6(D^2 + F^2) (2b_D + b_0) - 3(D + F)^2 (-2b_F + b_0) - 3(D - F)^2 (b_0 + 2b_f) , \\
\epsilon_{4,\Sigma}^{KK} &= 12(D^2 + F^2) b_0 - 6(D + F)^2 (b_0 + b_D + b_F) - 6(D - F)^2 (b_0 + b_D - b_F) , \\
\epsilon_{4,\Sigma}^{\eta\pi} &= \frac{2}{3}b_D , \quad \epsilon_{4,\Sigma}^{\eta K} = 0 .
\end{aligned} \tag{A.2}$$

$\Xi :$

$$\begin{aligned}
\epsilon_{1,\Xi}^{\pi} &= -4(4d_1 - 2d_5 + d_7 + 3d_8) , \quad \epsilon_{1,\Xi}^K = -16(d_1 + d_2 + d_3 + d_5 + d_7 + d_8) , \\
\epsilon_{2,\Xi}^{\pi K} &= 8(4d_1 + 2d_2 + d_5 - 2d_7 + 2d_8) , \\
\epsilon_{3,\Xi}^{\pi} &= 3(b_D - b_F + 2b_0 - b_1 + b_2 - b_3 - 2b_8) - \frac{3(D - F)^2}{4\overset{\circ}{m}} , \\
\epsilon_{3,\Xi}^K &= 2(3b_D + b_F + 4b_0 - 3b_1 - b_2 - 3b_3 - 4b_8) - \frac{5D^2 + 6DF + 9F^2}{6\overset{\circ}{m}} , \\
\epsilon_{3,\Xi}^{\eta} &= 2b_0 - 3b_1 - b_2 - \frac{1}{3}b_3 - 2b_8 - \frac{(D + 3F)^2}{12\overset{\circ}{m}} , \quad \epsilon_{4,\Xi}^{\pi\pi} = 0 , \quad \epsilon_{4,\Xi}^{\pi K} = 0 , \\
\epsilon_{4,\Xi}^{K\pi} &= (5D^2 + 6DF + 9F^2) (-2b_F + b_0) - \frac{9}{2}(D + F)^2 (2b_D + b_0) \\
&\quad - \frac{1}{6}(D - 3F)^2 (3b_0 - 2b_D) , \\
\epsilon_{4,\Xi}^{KK} &= 2(5D^2 + 6DF + 9F^2) (b_D + b_F + b_0) - 9(D + F)^2 b_0 \\
&\quad - \frac{1}{3}(D - 3F)^2 (3b_0 + 4b_D) , \\
\epsilon_{4,\Xi}^{\eta\pi} &= -b_D - \frac{5}{3}b_F , \quad \epsilon_{4,\Xi}^{\eta K} = \frac{8}{3}(b_D + b_F) .
\end{aligned} \tag{A.3}$$

B Renormalization of the kaon–nucleon σ –terms and scalar form factors

Here, we discuss the renormalization related to the two kaon–nucleon σ –terms and the three scalar form factors $\sigma_{\pi N}(t)$ and $\sigma_{KN}^{(1,2)}(t)$. Since we are dealing with composite operators, we use the standard procedure and generalize the original Lagrangian used to calculate the baryon masses, $\mathcal{L} = \sum_i \mathcal{L}_i$, in the following way

$$\mathcal{L} \rightarrow \mathcal{L} + \sum'_{q,i} c_{q,i} \left(-\frac{\partial \mathcal{L}_i}{\partial m_q} \right), \quad q = u, d, s, \quad (\text{B.1})$$

where the $c_{q,i}$ are sources and the prime on the sum indicates that not all values of q and i are necessarily taken. Consider now a diagram labelled 'j', which leads to a divergent contribution to the σ –term. The renormalization for that graph proceeds via

$$-\frac{\partial \mathcal{L}_i}{\partial m_q} \rightarrow -\frac{\partial \mathcal{L}_i}{\partial m_q} + \frac{1}{F_\pi^2} L \sum'_{k,q'} f_{q',k,j} \left(-\frac{\partial \mathcal{L}_k}{\partial m_{q'}} \right), \quad (\text{B.2})$$

with the $f_{q',k,j}$ are Clebsch–Gordan coefficients. To order q^4 , there are two types of divergences, namely $\sim L M^2 t$ and $\sim L M^4$. The t –dependent ones can be renormalized with the help of two counter terms of the form

$$\mathcal{L}_1 = \text{Tr}(\bar{B}\{\chi_+, [D_\mu, [D^\mu, B]]\}) , \quad \mathcal{L}_2 = \text{Tr}(\bar{B}[\chi_+, [D_\mu, [D^\mu, B]]]) . \quad (\text{B.3})$$

For the t –independent ones, one needs six independent terms,

$$\begin{aligned} \mathcal{L}_3 &= \text{Tr}(\bar{B}[\chi_+, [\chi_+, B]]) , \quad \mathcal{L}_4 = \text{Tr}(\bar{B}\{\chi_+, [\chi_+, B]\}) , \quad \mathcal{L}_5 = \text{Tr}(\bar{B}\{\chi_+, \{\chi_+, B\}\}) \\ \mathcal{L}_6 &= \text{Tr}(\bar{B}\chi_+) \text{Tr}(\chi_+ B) , \quad \mathcal{L}_7 = \text{Tr}(\bar{B}[\chi_+, B]) \text{Tr}(\chi_+) , \quad \mathcal{L}_8 = \text{Tr}(\bar{B}B) \text{Tr}(\chi_+) \text{Tr}(\chi_+) . \end{aligned} \quad (\text{B.4})$$

Consider as an example the composite operator $\bar{u}u$ and the corresponding matrix element $\langle p' | \bar{u}u | p \rangle$. A specific contribution to it is given by the diagram in fig. 2 with an insertion of the b_1 –vertex, leading to (cf. Eq.(31)), and neglecting for simplicity the $\pi^0 - \eta$ mixing,

$$I_\sigma(t) = i(4M_a^2 - t) L + \text{finite terms} \quad (\text{B.5})$$

Expanding the operator $\bar{u}u$ and the b_1 –term of the effective Lagrangian to second order in the meson fields, the divergent parts from I_σ contribute as follows

$$\langle p' | \bar{u}u | p \rangle_{\text{div}} = -\frac{8b_1}{F_\pi^2} B L \left(\frac{3}{2} M_\pi^2 + 2M_K^2 + \frac{1}{2} M_\eta^2 \right) + \frac{8b_1}{F_\pi^2} B L t . \quad (\text{B.6})$$

The t –dependent divergence is renormalized with the help of \mathcal{L}_1 via

$$-\frac{2b_1}{F_\pi^2} L \left\{ -3 \frac{\partial \mathcal{L}_1}{\partial m_u} - 2 \frac{\partial \mathcal{L}_1}{\partial m_d} - \frac{\partial \mathcal{L}_1}{\partial m_s} \right\} \quad (\text{B.7})$$

i.e . for the Clebsch–Gordan coefficients

$$f_{u,1,j} = -6 b_1 , \quad f_{d,1,j} = -4 b_1 , \quad f_{s,1,j} = -6 b_1 , \quad (\text{B.8})$$

and zero otherwise. The t -independent divergences in the matrix element $\langle p' | \bar{u}u | p \rangle$ are renormalized with the help of \mathcal{L}_3 , \mathcal{L}_5 , \mathcal{L}_6 and \mathcal{L}_8 ,

$$- \frac{b_1}{2 F_\pi^2} L \left\{ -\frac{5}{12} \frac{\partial \mathcal{L}_3}{\partial m_u} - \frac{5}{12} \frac{\partial \mathcal{L}_3}{\partial m_d} - \frac{9}{8} \frac{\partial \mathcal{L}_5}{\partial m_u} - \frac{5}{8} \frac{\partial \mathcal{L}_5}{\partial m_d} - \frac{1}{8} \frac{\partial \mathcal{L}_5}{\partial m_s} + 2 \frac{\partial \mathcal{L}_6}{\partial m_u} - \frac{1}{2} \frac{\partial \mathcal{L}_8}{\partial m_u} \right\} \quad (\text{B.9})$$

with the Clebsch–Gordan coefficients

$$\begin{aligned} f_{u,3,j} &= -\frac{5}{24} b_1 , \quad f_{d,3,j} = -\frac{5}{24} b_1 , \quad f_{u,5,j} = -\frac{9}{16} b_1 , \quad f_{d,5,j} = -\frac{5}{16} b_1 , \\ f_{s,5,j} &= -\frac{1}{16} b_1 , \quad f_{u,6,j} = b_1 , \quad f_{u,8,j} = -\frac{1}{4} b_1 . \end{aligned} \quad (\text{B.10})$$

C $\sigma_{\pi N}(0)$ and strangeness fraction coefficients at $\mathcal{O}(q^4)$

Here, we collect the coefficients $\epsilon_{i,\sigma}^{P(Q)}$ and $\epsilon_{i,y}^{P(Q)}$ with $i = 1, 2, 3$ and $P, Q = \pi, K, \eta$.

Pion–nucleon σ -term:

$$\begin{aligned} \epsilon_{1,\sigma}^\pi &= 2\epsilon_{1,N}^\pi + \frac{1}{2}\epsilon_{2,N}^{\pi K} + \epsilon_{3,N}^\pi + \epsilon_{4,N}^{\pi\pi} + \frac{1}{2}\epsilon_{4,N}^{\pi K} + \frac{1}{3}\epsilon_{4,N}^{\eta\pi} , \\ \epsilon_{1,\sigma}^K &= \epsilon_{1,N}^K + \epsilon_{2,N}^{\pi K} + \frac{1}{2}\epsilon_{3,N}^K + \frac{1}{2}\epsilon_{4,N}^{KK} + \epsilon_{4,N}^{\pi K} + \frac{1}{3}\epsilon_{4,N}^{\eta\pi} , \\ \epsilon_{1,\sigma}^\eta &= \frac{2}{3}\epsilon_{1,N}^\eta + \frac{1}{3}\epsilon_{3,N}^\eta + \frac{1}{3}\epsilon_{4,N}^{\eta\eta} , \quad \epsilon_{2,\sigma}^\pi = 2\epsilon_{3,N}^\pi + 2\epsilon_{4,N}^{\pi\pi} + \frac{1}{2}\epsilon_{4,N}^{\pi K} , \\ \epsilon_{2,\sigma}^K &= \epsilon_{3,N}^K + \epsilon_{4,N}^{K\pi} + \epsilon_{4,N}^{KK} , \quad \epsilon_{2,\sigma}^\eta = \frac{2}{3}\epsilon_{3,N}^\eta + \frac{2}{3}\epsilon_{4,N}^{\eta\eta} + \epsilon_{4,N}^{\eta\pi} + \frac{1}{2}\epsilon_{4,N}^{\eta K} , \\ \epsilon_{3,\sigma}^{K\pi} &= \epsilon_{4,N}^{\pi K} , \quad \epsilon_{3,\sigma}^{\pi K} = \frac{1}{2}\epsilon_{4,N}^{K\pi} , \quad \epsilon_{3,\sigma}^{\pi\eta} = \frac{1}{3}\epsilon_{4,N}^{\eta\pi} , \quad \epsilon_{3,\sigma}^{K\eta} = \frac{1}{3}\epsilon_{4,N}^{\eta K} . \end{aligned} \quad (\text{C.1})$$

Notice that the $\epsilon_{(2,3)\sigma}$ and the $\epsilon_{(3,4)N}$ include the factor $1/\Lambda_\chi^2$ which is not made explicit here.

Strangeness fraction:

$$\begin{aligned} \epsilon_{1,y}^\pi &= \frac{1}{2}\epsilon_{2,N}^{\pi K} + \frac{1}{2}\epsilon_{4,N}^{K\pi} + \frac{2}{3}\epsilon_{4,N}^{\eta\pi} , \quad \epsilon_{1,y}^K = \epsilon_{1,N}^K + \frac{1}{2}\epsilon_{3,N}^K + \frac{2}{3}\epsilon_{4,N}^{\eta K} , \\ \epsilon_{1,y}^\eta &= \frac{4}{3}\epsilon_{1,N}^\eta + \frac{2}{3}\epsilon_{3,N}^\eta , \quad \epsilon_{2,y}^\pi = 0 , \quad \epsilon_{2,y}^K = \epsilon_{3,N}^K \\ \epsilon_{2,y}^\eta &= \frac{4}{3}\epsilon_{3,N}^\eta + \frac{1}{2}\epsilon_{4,N}^{\eta K} , \quad \epsilon_{3,y}^{\pi K} = \frac{1}{2}\epsilon_{4,N}^{K\pi} , \quad \epsilon_{3,y}^{K\pi} = 0 , \quad \epsilon_{3,y}^{\pi\eta} = \frac{2}{3}\epsilon_{4,N}^{\eta\pi} , \quad \epsilon_{3,y}^{K\eta} = \frac{2}{3}\epsilon_{4,N}^{\eta K} . \end{aligned} \quad (\text{C.2})$$

D Determination of the Roper–octet coupling constants D_R and F_R

The only listed decays to determine the coupling constants D_R and F_R are the $N^*(1440) \rightarrow N\pi$ and the $\Lambda^*(1600) \rightarrow \Sigma\pi$. Consider first the Λ^* decay. The width follows via

$$\Gamma = \frac{1}{8\pi m_R^2} |\vec{q}_\pi| |\mathcal{T}|^2 \quad , \quad (\text{D.1})$$

with $|\vec{q}_\pi| = 334$ MeV the pion momentum in the restframe of the Λ^* and we use the spinor normalization $\bar{u}(p)u(p) = 2m_R$. Straightforward calculation gives

$$\begin{aligned} \Gamma &= \frac{D_R^2}{16\pi m_{\Lambda^*} F_\pi^2} |\vec{q}_\pi| \left[2E_\pi (E_\Sigma E_\pi + \vec{q}_\pi^2) - M_\pi^2 (E_\Sigma + m_\Sigma) \right] \\ &= 149 \text{ MeV } D_R^2 \quad , \end{aligned} \quad (\text{D.2})$$

with $E_\pi = (M_\pi^2 + \vec{q}_\pi^2)^{1/2}$ and $E_\Sigma = (m_\Sigma^2 + \vec{q}_\pi^2)^{1/2}$. From the PDG value $\Gamma(\Lambda^* \rightarrow \Sigma\pi) = (150 \pm 100) \text{ MeV} \cdot (0.35 \pm 0.25) = 52.5 \text{ MeV}$ we derive

$$D_R = 0.60 \pm 0.41 \quad . \quad (\text{D.3})$$

The sign of D_R is, of course, not fixed. We will chose it to be positive in accordance with the ground state octet D coupling. The $N^*(1440)N\pi$ effective Lagrangian is

$$\begin{aligned} \mathcal{L}_{N^*N\pi} &= \frac{g_A}{4} \sqrt{R} \bar{\Psi}_{N^*} \gamma_\mu \gamma_5 u^\mu \Psi_N + \text{h.c.} \quad , \\ g_A \sqrt{R} &= D_R + F_R \quad . \end{aligned} \quad (\text{D.4})$$

Here, $\sqrt{R} = 0.53 \pm 0.04$ and $g_A = 1.33$ as given by the Goldberger–Treiman relation. As explained in some detail in Ref.[41], we use the width of the N^* determined from the speed plot, not the the model–depedent Breit–Wigner fits, $\Gamma_{\text{tot}} = 160 \pm 40 \text{ MeV}$. Assuming F_R to be positive (as is the equivalent hyperon coupling), we find by using Eq.(D.3),

$$F_R = 0.11 \mp 0.41 \quad . \quad (\text{D.5})$$

These are the values of the Roper couplings used in the main text.

References

- [1] P. Langacker and H. Pagels, Phys. Rev. **D8** (1971) 4595
- [2] H. Pagels, Phys. Rep. **C16** (1975) 219
- [3] J. Gasser, Ann. Phys. **136** (1981) 62
- [4] J. Gasser and H. Leutwyler, Phys. Rep. **C87** (1982) 77
- [5] E. Jenkins, Nucl. Phys. **B368** (1992) 190
- [6] E. Jenkins and A.V. Manohar, in “Effective Field Theories of the Standard Model”, ed. Ulf-G. Meißner, World Scientific, Singapore, 1992
- [7] V. Bernard, N. Kaiser and Ulf-G. Meißner, Z. Phys. **C60** (1993) 111
- [8] R.F. Lebed and M.A. Luty, Phys. Lett. **B329** (1994) 479
- [9] S. Mallik, Z. Phys. **C68** (1995) 149
- [10] M.K. Banerjee and J. Milana, Phys. Rev. **D52** (1995) 6451
- [11] S. Dentin, Thesis, Université d’Aix–Marseille II, 1990 (unpublished)
- [12] U. Bürgi, Diploma Thesis, University of Bern, 1994 (unpublished)
- [13] A. Kallen, “The $m^4 \ln m$ Contribution to the Nucleon Mass in CHPT”, Bern University preprint BUTP–96/13, 1996 [hep-ph/9604205].
- [14] E. Jenkins and A.V. Manohar, Phys. Lett. **B255** (1991) 558
- [15] T.R. Hemmert, B.R. Holstein and J. Kambor, “Systematic $1/m$ expansion for spin $3/2$ particles in baryon chiral perturbation theory”, IPN Orsay preprint IPNO-TH-95-60 [hep-ph/9606456]
- [16] B. Borasoy and Ulf-G. Meißner, Phys. Lett. **B365** (1996) 285
- [17] S. Weinberg, Physica **96A** (1979) 327
- [18] J. Gasser and H. Leutwyler, Ann. Phys. **158** (1984) 142
- [19] J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 465
- [20] J. Gasser and H. Leutwyler, Nucl. Phys. **B94** (1975) 269
- [21] E. Jenkins and A.V. Manohar, Phys. Lett. **B281** (1992) 336
- [22] J. Gasser, H. Leutwyler and M.E. Sainio, Phys. Lett. **B253** (1991) 252, 260

- [23] M.E. Sainio, in “Chiral Dynamics: Theory and Experiment”, A.M. Bernstein and B.R. holstein (eds.), Springer, Heidelberg, 1995.
- [24] Ulf-G. Meißner, Rep. Prog. Phys. **56** (1993) 903
- [25] G. Ecker, Prog. Nucl. Part. Phys. **35** (1995) 1
- [26] V. Bernard, N. Kaiser and Ulf-G. Meißner, Int. J. Mod. Phys. **E4** (1995) 193
- [27] A. Pich, Rep. Prog. Phys. **58** (1995) 563
- [28] V. Bernard, N. Kaiser, J. Kambor and Ulf-G. Meißner, Nucl. Phys. **B388** (1992) 315
- [29] J. Bijnens et al., Phys. Lett. **B374** (1996) 210
- [30] B. Borasoy, Thesis, University of Bonn, in preparation.
- [31] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321** (1989) 311
- [32] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. **B223** (1989) 425
- [33] J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. **D39** (1989) 1947
- [34] V. Bernard, N. Kaiser, Ulf-G. Meißner and A. Schmidt, Phys. Lett. **B319** (1993) 269
- [35] M.N. Butler and A.M. Nathan, in “Chiral Dynamics: Theory and Experiment”, A.M. Bernstein and B.R. Holstein (eds.), Springer, Heidelberg, 1995.
- [36] J. Gasser, M.E. Sainio and A. Svarc, Nucl. Phys. **B307** (1988) 779
- [37] A. Schmidt, Thesis Technische Universität München, 1994 (unpublished).
- [38] V. Bernard, N. Kaiser, Ulf-G. Meißner and A. Schmidt, Z. Phys. **A348** (1994) 317
- [39] V. Bernard, N. Kaiser and Ulf-G. Meißner, Z. Phys. **C70** (1996) 483; Phys. Lett. **B378** (1996) 337
- [40] M.K. Banerjee and J. Milana, “The Decuplet Revisited in χ PT”, preprint DOE-ER-40762-065, 1994 [hep-ph/9508340]
- [41] V. Bernard, N. Kaiser and Ulf-G. Meißner, Nucl. Phys. **B457** (1995) 147
- [42] P.B. Siegel, N. Kaiser and W. Weise, Nucl. Phys. **A594** (1995) 325
- [43] N. Kaiser, T. Waas and W. Weise, “SU(3) chiral dynamics with coupled channels: eta and kaon photoproduction”, preprint TUM/T39-96-20, 1996

- [44] J.F. Donoghue, B.R. Holstein and D. Wyler, Phys. Rev. **D47** (1993) 2089;
R. Urech, Nucl. Phys. **B433** (1995) 234;
H. Neufeld and H. Rupertsberger, Z. Phys. **C68** (1995) 91;
R. Baur and R. Urech, Phys. Rev. **D53** (1996) 6552
- [45] J. Kambor, private communication
- [46] A.I. Daydychev et al. , Nucl. Phys. **B397** (1993) 123
- [47] J. Bijnens et al. in “The DAΦNE Handbook”, second edition, L. Maiani, G. Pancheri and N. Paver (eds.) , INFN Frascati, 1995
- [48] M.N. Butler, M.J. Savage and R.P. Springer, Nucl. Phys. **B399** (1993) 69
- [49] M.A. Luty and M. White, Phys. Lett. **B319** (1994) 261

Tables

B	λ_B [GeV ²]	D_B^β [GeV ²]	D_B^δ [GeV ⁴]	D_B^ϵ [GeV ²]
N	0.030	(−1.165,+0.233)	(−0.123,−0.079)	(0.114,0.029)
Λ	0.065	(−2.013,−0.783)	(+0.077,−0.053)	(0.219,0.035)
Σ	0.058	(−3.626,−0.631)	(−0.802,−0.186)	(0.506,0.058)
Ξ	0.096	(−3.205,−1.648)	(−0.390,−0.108)	(0.471,0.052)

Table 1: Numerical values of the state-dependent coefficients in Eq.(59). The first and second term in the brackets refers to the decuplet and Roper-octet contribution, respectively.

	b_0	b_D	b_F	b_1	b_2	b_3	b_8
$\beta_R = 0$	−0.606	0.079	−0.316	−0.004	−0.187	+0.018	−0.109
Ref.[42]	−0.493	0.066	−0.213	+0.044	−0.145	−0.054	−0.165
Ref.[7]	−0.750	0.016	−0.553	-	-	-	-

Table 2: Low-energy constants from $\mathcal{L}_{\phi B}^{(2)}$ in GeV^{−1} for $\beta_R = 0$.

λ [GeV]	b_0 [GeV ⁻¹]	b_D [GeV ⁻¹]	b_F [GeV ⁻¹]	$\overset{\circ}{m}$ [MeV]	$m_N^{(2)}$ [MeV]	$m_N^{(4)}$ [MeV]	$\hat{\sigma}$ [MeV]	y
0.8	-0.643	0.085	-0.369	849.	244.	119.	49.4	-0.10
1.0	-0.606	0.079	-0.316	767.	261.	183.	35.6	+0.21
1.2	-0.574	0.072	-0.281	613.	266.	232.	25.9	+0.42

Table 3: Theoretical uncertainties due to the renormalization scale λ . For comparison, we give the second and fourth order contribution to the nucleon mass, $m_N^{(2)}$ and $m_N^{(4)}$, respectively. The third order contribution is $m_N^{(3)} = -272$ MeV. We set $\beta_R = 0$.

$\sigma_{\pi N}(0)$ [MeV]	b_0 [GeV ⁻¹]	b_D [GeV ⁻¹]	b_F [GeV ⁻¹]	$\overset{\circ}{m}$ [MeV]	$m_N^{(2)}$ [MeV]	$m_N^{(4)}$ [MeV]	$\hat{\sigma}$ [MeV]	y
35	-0.508	0.082	-0.319	910.	163.	139.	37.4	-0.07
45	-0.606	0.079	-0.316	767.	261.	183.	35.6	+0.21
55	-0.691	0.074	-0.310	625.	360.	226.	37.1	+0.34

Table 4: Theoretical uncertainties due to the pion-nucleon σ -term. For comparison, we give the second and fourth order contribution to the nucleon mass, $m_N^{(2)}$ and $m_N^{(4)}$, respectively. The third order contribution is $m_N^{(3)} = -272$ MeV. We set $\beta_R = 0$.

Figure captions

Fig.1 One-loop graphs with exactly one insertion from $\mathcal{L}_{\phi B}^{(2)}$ (circlecross). Goldstone boson renormalizations are not shown.

Fig.2 Contribution to the scalar form factor. The double-line denotes the insertion of $\hat{m}(\bar{u}u + \bar{d}d)$.

Fig.3 Baryon resonance excitation involving pion loops. The double-line represents the decuplet or the even-parity Roper octet. Solid and dashed lines denote the ground state octet baryons and Goldstone boson fields, respectively.

Fig.4 Scalar meson excitation. The double-line represents the scalars and the circlexcross a quark mass insertion $\sim \chi_+$.

Fig.5 Baryon resonance excitation: Two-loop graphs. The double-line represents the decuplet or the even-parity Roper octet or a combination thereof. Solid and dashed lines denote the ground state octet baryons and Goldstone boson fields, respectively. Graphs of the same topologies with one or two groundstate propagators are not shown.

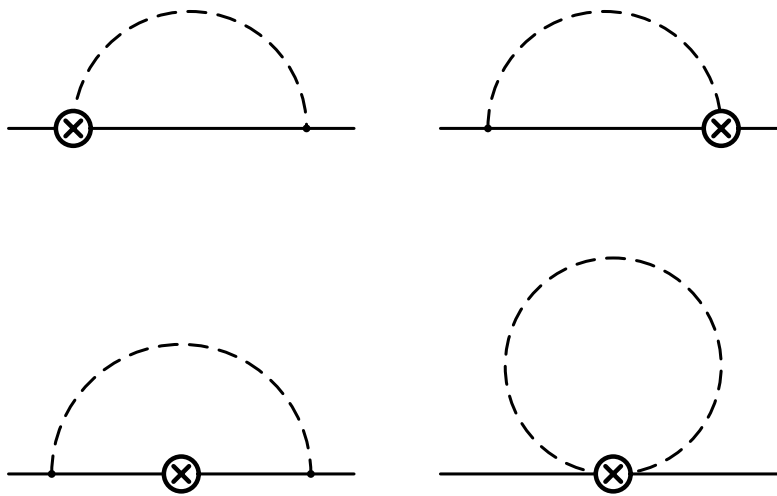


Figure 1

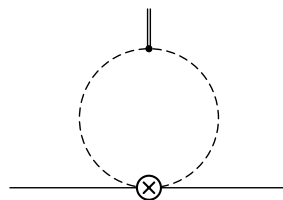


Figure 2

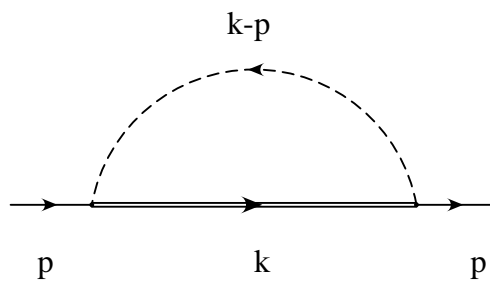


Figure 3

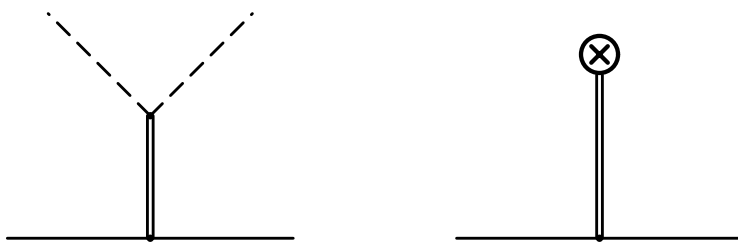


Figure 4

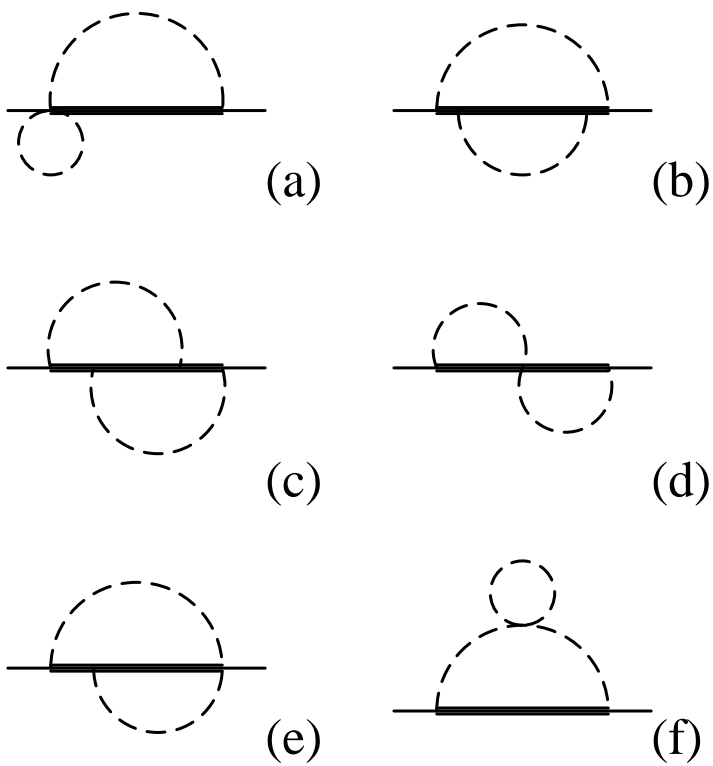


Figure 5